

DOCUMENT RESUME

ED 426 256

CE 077 938

TITLE Math Skills Curriculum Binder. Workplace Training Project.
INSTITUTION Lane Community Coll., Eugene, OR.
SPONS AGENCY Office of Vocational and Adult Education (ED), Washington, DC. National Workplace Literacy Program.
PUB DATE 1997-00-00
NOTE 327p.; For the "Language and Communication Skill Curriculum Binder", see CE 077 937. The other partners in The Workplace Training Project are: Newood Products, Pacific Western Extruded Pipe Company, Springfield Forest Products, Staffing Services, HMT Technology.
PUB TYPE Guides - Classroom - Teacher (052)
EDRS PRICE MF01/PC14 Plus Postage.
DESCRIPTORS Adult Basic Education; Adult Literacy; Algebra; Arithmetic; Basic Skills; Behavioral Objectives; Calculators; Classroom Techniques; Community Colleges; *Competency Based Education; Curriculum Guides; Fractions; *Functional Literacy; Geometry; Learning Activities; Lesson Plans; *Literacy Education; Mathematical Formulas; *Mathematics Skills; Measurement Techniques; Numeracy; *Partnerships in Education; Performance Based Assessment; School Business Relationship; Two Year Colleges; *Workplace Literacy
IDENTIFIERS *Job Related Mathematics

ABSTRACT

This document, which is intended for workplace trainers, contains materials for conducting five workplace mathematics courses that were developed through the Workplace Training Project, a partnership involving Lane Community College in Oregon and five area businesses. The five courses, which were developed by project staff based on the business partners' input regarding the mathematics skills required of employees at their specific worksites, were presented to 419 students. The course titles are as follows: basic math; fractions and measurement in the workplace; fractions/decimals/formulas in a workplace setting; calculators in the workplace; and geometry/algebra. The following materials are included for each course: course description; student outcome objectives; description of target student population; description of class environment; suggested instructional techniques; syllabus of course activities; learning activity sheets; student handouts; student worksheets; sample problems; and substantial bibliography. Each learning activity sheet contains the following: introductory activities; list of resources and materials needed; description of the instruction process; performance assessment methods and results; and comments. (MN)

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Workplace Training Project

**Math
Skills**

Curriculum Binder

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is a partnership with
Lane Community College and
Newood Products
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Workplace Training Project

Math Skills

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ACKNOWLEDGEMENTS

The Workplace Training Project, funded by a National Workplace Literacy grant, brought together Lane Community College and five businesses: Newood Products, PW Pipe, The Springfield Group, Staffing Services, and HMT Technology. As a part of our grant, project staff wrote curriculum in response to training needs at each of the businesses. Business partners assisted staff in identifying important objectives and in supplying necessary information and documentation for trainings. As a result of these combined efforts, the project developed two curriculum guides: language and communication, and math.

Elaine Pray, project coordinator, would like to thank the efforts of project staff and business partners who worked on creating curriculum for the project trainings, and most importantly, the 419 employees who participated in the trainings.

Business Partners:

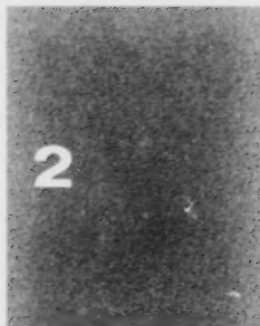
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Basic Math



Fractions and Measurement in the Workplace



Fractions / Decimals / Formulas in a Workplace Setting



Calculators in the Workplace



Geometry/Algebra

Workplace Training Project

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Course Description:

This basic math class took place at a manufacturing plant. Four classes met at different times of day, but during the same period of weeks. The instructor, in cooperation with the continuous improvement coordinator and the site advisory team, developed a customized math screening test based on the math concepts, computations, and applications actually used at this facility. This test was required of all production personnel; some employees from shipping and other job categories also took the test voluntarily. The instructor went over the test scores with everyone who took the test. She helped them interpret the scores and discussed with them their need for and interest in improved math skills. She also made recommendations to help them meet their needs, especially whether or not they should take this basic math class. For those who did not need this basic math class, the instructor made note of needs and interests to address in future classes. The participants in the basic math class are those who needed a lot of work on basic math skills as defined by the test.

The purpose of this course was to develop the ability of employees to understand and use the math relevant to this production environment. The course content had four main components:

- 1) Math concepts and number relationships. This included such ideas as place value, number patterns, estimation, as well as developing an understanding of the meaning of fractions, decimals, ratios, percents.
- 2) Computation review, especially with whole numbers and decimals. This also included fraction/decimal conversion and some calculator information and practice.
- 3) Ratios, proportions, and percents. Many work situations involve understanding and applying proportions and percents. The instructor teaches percents as an extension of ratio and proportion.
- 4) Application, application, application.

From her work developing the screening test and further discussion with participants and others, the instructor developed a set of workplace math applications. These applications have a bearing throughout the course but particularly during the later sessions.

The original organizational structure of the class had the first 5 weeks devoted to basic concepts, calculation, and relationships. After a two week break for Christmas, the remaining 6 weeks were to focus on practical applications with one session for closing and paperwork. In fact, it required significantly more time to build understanding. The class followed this basic plan, but was also

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extended for an additional 5 weeks (a total of 17 weeks). The instructor continually integrated work on understanding concepts and number relationships into the work with practical applications.

Student Outcome Objectives

Participants will demonstrate the ability to:

1. Use math concepts and number relationships
2. Compute with whole numbers and decimals
3. Estimate and learn to recognize a reasonable answer
4. Convert fractions, decimals, and percents
5. Work with ratios and proportions
6. Work with percents
7. Demonstrate practical applications on the job and off

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Student Population:

Most participants were production workers; about one-fourth had supervisory roles. Also, a few participants were from shipping or elsewhere in the plant. One class had only two participants in it with a supervisor and assistant supervisor. The largest class had 6 participants in it. One class required particularly tactful instruction because a supervisor in that class had very low math skills. Having supervisors and line workers in classes together made for some awkward moments in the beginning, as a supervisor might be hesitant to speak out and risk his authority or reputation by exposing low skills. A line worker might feel his or her job was in jeopardy if his or her supervisor knew how low their skills were. The instructor tried to create an environment of "we're all here to learn together, from each other." As the class continued, dialog and questions both flowed more freely. That was one indication that the level of trust and comfort improved.

Class Environment:

Because the initial screening test was mandatory, some participants felt coerced into taking the class. Participants did not feel they needed basic math skills to do their job, but feared being fired if they didn't take it. Management did not present the class in that light. In fact, they stated quite clearly that their interest was in improving skills, not in firing anyone for lack of skills. Even so, there was a lot of friction over why management expected them to take the class. Some, though not all, initially resistant participants seemed to come to acceptance and even appreciation of the class. Examples of acceptance included requests for computer time to practice, and an increased demonstration of self-confidence.

The class met in the company conference room. It sometimes seemed a little large for the smaller classes, but generally it worked quite well. It had comfortable seating, plenty of tables, an overhead, white board, and space to put up posters. The original design was a 1 1/2 hour class, but scheduling conflicts necessitated shortening it to one hour and twenty minutes. Each class had participants from a particular shift. The class met either before the participants' shift or after their shift. For the most part, everybody in the class worked with each other on the job.

Instructional Techniques: ✓

☐ Multi-media	✓ Small group	✓ Cooperative learning
✓ Lecture	✓ Computer-aided	✓ Assignments
✓ Individualized instruction	☐ Simulation	✓ Peer tutoring
☐ Role-playing	☐ Project	✓ In-class practice
Other: For a portion of the class, participants had access to a basic math computer program which some of them used for additional practice. They also got help from each other (peer tutoring, but out of class)		

Basic Math

Syllabus of Course Activities:

- I. Introductory Activities
- II. Whole Number Concepts, Number Relationships and Computation
- III. Understanding Decimals: Place Value, Comparisons, and other Relationships
- IV. Working with Decimal Numbers: Estimation, Computation, and Word/Work Problems
- V. Fraction-Decimal Conversion
- VI. Working with Ratios and Proportions
- VII. Working with Percents
- VIII. Practical Applications On the Job and Off
- IX. Concluding Activities

Activity I: Introductory Activities

1. Grant paperwork and pretest
2. Review of objectives to be achieved in class and what they mean
3. Learning styles/learning channel exercise and discussion
4. Add any additional objectives participants want covered in the class
5. Preparation of Individualized Learning Plan (ILP)
6. "*Math Training Preview*"
7. Introduction of "*Math Practice Log*"
8. Introduction of "*Math Anxiety Bill of Rights*" and class "*Ground Rules*"

Resources and Materials Needed:

- Grant paperwork, including ILP with class objectives for participants to review
- # "*Math Training Preview*" - a list of questions about math background and preferences Mary Mayfield, Instructor, Lane Community College, 1996.
- Checklist of math skills to be covered in this class (See comments section.)
- # "*Math Practice Log*" Mary Mayfield, 1996.
- # "*Math Anxiety Bill of Rights*" Mary Mayfield, 1996.
- # "*Ground Rules*" Mary Mayfield, 1996.

Description of Instruction:

- Did all the grant paperwork described above as required, including filling out the first part of the ILP.
- "*Math Training Preview*" was one way for the instructor to gather some information about participants that would help her customize the class based partly on their skills, interests, and experience.
- The instructor gave each participant a "*Math Practice Log*" and asked them to check the appropriate box for each "math interaction" they had during the week. The purpose of the math log was to give participants and the instructor an opportunity to recognize how they used math outside of class.

- This mark indicates a copy of the handout is in the **Handout** section of this curriculum.

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- When the instructor developed the pretest, she broke down the skills the company wanted the employees to have into a Checklist of about 25 items. She put these on a grid to be used to track what participants had worked on and when they accomplished competency. The list was handed out as introductory material. The instructor discussed it with participants so they would understand what skills they could expect to learn in the class. Participants kept their lists in their folders and gave them to the instructor at the beginning of class. She, then, tried to note on the grid any progress or show of competence she noticed in that class session. The object was to give participants graphic feedback on their progress and the skills they were demonstrating in class.
- The instructor gave each participant a "*Math Practice Log*" and asked them to check the appropriate box for each "math interaction" they had during the week. The purpose of the math log was to give participants and the instructor an opportunity to recognize how they used math outside of class.
- After giving participants "*Math Anxiety Bill of Rights*" and "*Ground Rules*," the instructor allowed participants some time to read through them and to ask questions.

Performance Assessment Methods and Results:

- Completion of forms and individualized learning plans. The participants need to know the subjects covered during the class, and that this class will meet their needs. Reviewing the objectives and creating an ILP achieves that goal with the participants.
- Presentation and discussion of "*Math Anxiety Bill of Rights*" and "*Ground Rules*" was designed to establish both the factual (food and drink is OK; bring your materials to class) and the attitudinal (all people and ideas are respected; you have the right to dislike math) contexts of the class. Sometimes discussion was varied and lively; sometimes it was minimal. In either case, participants knew what to expect (and what was expected of them) in the class.

Comments:

The Checklist of Skills was a good idea, but hard to implement, mostly because the instructor found herself always involved in instruction and not having/allowing the time to fill in the form. Also, she was unclear about just how to fill it in. At first, she just used a check mark for "showing competence," but found that not definitive enough; then she tried adding a date of "final competence" but this, too, wasn't really satisfactory. Finally, she used a scale of 1 (this material was presented in class) to 5 (They've got it). This was actually easier to use and gave more information than the previous attempts. The Checklist seems still to have potential for this daily tracking sort of use, but the "bugs" aren't entirely worked out. It did seem useful, though, in that it kept the specific skill objectives always available to participants. Perhaps its most valuable use was as a modified version with lines for instructor comments. The instructor gave this sheet to participants at the end of the course (along with their posttest results) as a summary of what their competencies were as shown by their daily work and the posttest. This is the Checklist version titled "*Math Competencies*" included with this curriculum.

Activity II: Whole Number Concepts, Number Relationships and Computation

1. Introduce, discuss, emphasize, and experiment with concepts and number relationships to give participants both a "bigger picture" and some understanding of the structure math is based on (how numbers work).
2. Reestablish computation skills with whole numbers.
3. Begin to transition into working with the calculator.

Resources and Materials Needed:

- Appropriate workbook pages or worksheets for times table review and computation practice
- "Characteristics of Today's and Tomorrow's Workplace" Excerpt from What Work Requires of Schools: A SCANS Report for America 2000, U.S. Department of Labor. Secretary's Commission on Achieving Necessary Skills, Washington DC, 20210, 1990.
- # "Open Calculation" Mary Mayfield, Instructor, Lane Community College, 1996.
- # "Number Patterns, Problem-Solving, and Review" Mary Mayfield, 1996.
- # "Finding Averages" Mary Mayfield, 1995.

Description of Instruction:

1. The class worked on some warm-up practice problems written on the white board, for example, $9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 0 =$, and see what participants do with it. Generally more than half the class calculates and a couple of people say, "Oh, zero!" The class then talks about the properties of zero and other numbers (such as 1). They also talk about the importance of study skills such as reading through an entire problem before doing anything, paying attention to detail, etc.

Another warm-up problem was: "a student divided a number by six instead of multiplying, he obtained sixteen as a quotient. What would his answer have been if he multiplied?"

This is a way of learning how much participants understand about operations with whole numbers.

Can they think about multiplying and dividing as inverse operations?

What happens if you do the wrong one?

This is an also an opportunity for participants to begin thinking and talking about how to approach solving word problems.

2. The instructor gave "Characteristics of Today's and Tomorrow's Workplace" to participants as a background for why they needed to learn some of the basic math skills. The handout talks about how many companies are moving toward a more team-oriented approach to doing business. It says that workers are being given more authority and that they, therefore, need to be multi-skilled. It talks about how more businesses are utilizing

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work teams. This was a way of putting this class and the importance of basic math knowledge in a current workplace context.

3. Everyone did a quick times table review. The instructor asked them to think only a second or two and write the answers that come. "If it doesn't come, then skip it." The instructor wanted to see which facts participants knew rather than which ones they could figure out. Participants have a hard time with that. They want to think about it and make sure they get it right!
4. The class did a review of addition and subtraction to make sure participants could do that. Participants were generally pretty good at these skills, but some needed to review borrowing.
5. The class practiced multiplying and dividing whole numbers. The instructor did some sample problems on the board as participants gave her instructions.
6. Along with all the computational practice, the instructor introduced a concept and number relation piece called "*Open Calculation*." Each problem on this worksheet includes enough information for someone to actually find all the numbers, but you have to understand and think through the process, as well as work both "backwards" and "forwards" to solve the problems. For instance, in solving an open addition problem you may have to both add and subtract at different times. You have to understand place value and the effect of carrying over amounts from previous columns, etc.

For the problems that include letters (such as $y + y + y = my$), some participants are initially confused -- "How can a letter be a number? What are we doing with letters in a math problem?" The instructor may ask some clarifying questions to help them visualize what is intended.

Again, the instructor's goal was to present them with something unusual and ask them to use familiar computational processes, operations, properties and patterns to try to make sense of computation in this new context. Participants often are not used to analyzing or doing things step-by-step. The instructor is trying to give them practice in thinking both systematically and mathematically. This helps them make sense of a new situation and analyze how to carry over what they already know into working with something unfamiliar.

7. The class did some mental math as another way of looking at number properties, patterns and operations. The instructor used this exercise to make computations easier and quicker. For example, when you are adding a column of numbers, you might first add the pairs that add up to ten, then add in the rest. This concept was expanded to look for other familiar sets of numbers--pairs that add to 100, for instance, or multiples of 25 (since we use these so handily in working with money).

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In a technique called halving and doubling, you double one number and halve the other, until you arrive at an operation that is easy to do mentally. For example:

if you are multiplying 17×16 , you double 17, take half of 16, and get 34×8
then you get 68×4
then 136×2 which equals 272

Each of these techniques works better for some problems than others, so you have to choose a strategy. Sometimes it's obvious what would be helpful; sometimes not. The class also practiced multiplying and dividing easily by 10s 100s and 1000s. Most of this was done in an informal way such as putting a number on the board and asking how you would do this. Participants would look for an easy way, or the instructor would say, "sometimes I do this," and so on. Just a dialog. The class didn't do a worksheet on these mental techniques, but the instructor came back to them as appropriate throughout the class.

8. At the beginning of each class the instructor provided time for participants to fill out their "Math Practice Log."
9. "Number Patterns, Problem-Solving, and Review" is a collection of rather unusual problems involving only whole numbers. Most of the problems have simple approaches but seem difficult at first reading. The purpose of the worksheet is:
 - a) to revisit number concepts and vocabulary such as "digit," "divisible," reading large numbers, etc.
 - b) to try out some problem-solving techniques such as recognizing what you know, finding a place to start, drawing a picture, or "guess and check"
 - c) to build confidence by illustrating how complicated-looking problems can be solved with basic math skills and simple steps
 - d) to become aware of and begin to verbalize their problem-solving processes
 - e) have a little fun

The instructor prefaced giving out the worksheet by telling the participants that it was a bit weird, but was designed to be fun, and she hoped they would approach it in that spirit. For homework, she asked them to read through and find two problems of different types (some were similar) to explore and work out. Participants had to be ready to explain to the class how they approached the problem. The class discussed the problems, put them up on the overhead and talked about what the process was they used to solve the problem. Participants also discussed what other processes could be used to solve the problem, and talked about the number relationships and concepts.

"Number Patterns, Problem-Solving, and Review" was given out as homework again. Participants were to find a problem not done in class (this limited them to 4 or 5 problems), and prepare to come back to class to talk about it. One of the skills the company wanted was for participants to be able to communicate ideas, process, or instructions. The instructor used this worksheet to help participants to think about their

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process and be able to explain it to someone else. Sometimes participants wrote their thinking steps, then verbalized them; at other times they just verbalized their explanations.

10. The instructor introduced averages by asking each participant for some piece of information:

how old they are,

how much money they might have in their possession

how long they have worked at the company, etc.

Then the instructor asked the class how to find their average age, etc. Usually, the class would then practice a few basic finding average problems from worksheet or workbook pages. Then the instructor handed out "*Finding Averages*." This worksheet begins with a couple of straight averaging problems, but is designed to get participants to think about averages and work with the concept of averages in many different contexts. It was quite challenging for many participants and required a lot of instructor support.

Performance Assessment Methods and Results:

- In the second warm-up activity (Description of Instruction, #1), most participants were unable to think about multiplication and division as inverse operations, and that gave the instructor a good idea of where to begin teaching.
- The "*Math Practice Log*" provided a good opportunity for both the instructor and the participants to see what participants were doing with their math skills during the time between classes.
- The instructor did not review on paper very much of what participants did. A lot of participants felt that unless the teacher recorded a score, they didn't learn. The instructor felt that a change in this thinking was an important part of "Learning to Learn." It took real effort on the instructor's part to help participants understand that the value is in the learning. The instructor was getting and giving feedback with whatever they do. Participants were giving each other feedback, whether the instructor checked every problem or not. Participants had been trained that the only way to see if they are learning is to look for the red marks. In this class the focus was on learning the process.

Comments:

- Participants were basically familiar with whole number concepts and computations. Most seemed competent with these skills after brief review, although division was still slow and awkward for some people. The class spent part of the introductory session, a whole class, and part of the third session (to go over the homework) on this section.
- Participants have been VERY cooperative in these classes. The instructor was pleased (if a little surprised) at how willing participants were to do whatever they were asked, even if some of it seemed a little "off the wall" to them.

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- All of the classes worked together a little bit. One class worked and learned especially well together. That one class had four participants with very basic skills, and one with pretty good skills (skills that he has mostly learned on his own, so they are not entirely reliable). Usually he could answer questions quickly, but did not know how he got there. This participant is a supervisor, but he did not seem to intimidate anyone. However, sometimes other class members were left out of the discussion by his quick responses. The instructor talked some with him about this, and she found work for him to do on his own. He could participate fully in the class when it seemed beneficial and could add to his knowledge in other ways when that seemed most appropriate. The other four participants interacted a lot by helping each other out. All of them asked questions and would turn to the person next to them for help. It was wonderful to watch them helping each other because each of them gained confidence in what they had learned and improved their ability to explain their ideas and processes to others.
- Having classes composed of participants from the same shift was effective because it improved their communication and willingness to help each other on the job. Things should go better on the job if participants continue working with each other on common problems.
- Participants noticed that the instructor always asked HOW they approach a problem and WHY they chose that approach. From the instructor's point of view, one of the main things the participants were learning is how to think about what it is they are doing. She always asked:

“How did you do that?”

“Why did you do that?”

“What were you thinking?”

“What did you do first?”

As a result, participants begin thinking about how to approach a problem instead of trying only to memorize procedures.

Activity III: Understanding Decimals: Place Value, Comparisons, and Other Relationships

1. Emphasis on visualization, clarification, and description of decimals and understanding decimal concepts
2. Practice with place value and decimals
3. Practice with number lines and other ways of visualizing decimal numbers
4. Practice with comparison of decimal numbers and ordering decimal numbers by size

Resources and Materials Needed:

- # Grid paper with multiple 10 x 10 squares for each participant, overhead of the grid and an overhead projector
- Base ten blocks
- # "Number-Line Decimals", Problem Solving in Mathematics, Lane County Mathematics Project, Grade 5, p. 197, 1983, Dale Seymour Publications.
- # "Decimals" grid from Personalized Computational Skills Program: Module C, Skill 1: pg. 1, Bryce R. Shaw, Houghton Mifflin Company, 1980.
- # "Decimal Sense" David Shellabarger, Instructor, Lane Community College, 1995.
- # "Decimal Sense--Place Value" David Shellabarger, 1995.
- # "Going in Circles" Problem Solving in Mathematics, Lane County Math Project, 4th Grade Book, 1983.
- # "Lineup with Decimals", Have Fun with Math, Louis Grant Brandes, J. Weston Walch, Portland, Maine 1988.

Description of Instruction:

1. To clarify the concept of decimals the instructor began by working on place values.

The instructor drew on the white board a long series of numbers with a decimal point in the middle (such as 3,492,204,147.27098). She then led a discussion that emphasized the main characteristics of decimal numbers:

- The purpose of the decimal point is to separate the whole numbers from the decimal fractions.
- Numbers to the left of the decimal point are whole numbers; numbers to the right of the decimal point represent parts of a whole
- There can be no whole numbers to the right of the decimal point.

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This was a discussion approach to the subject, with the instructor laying out the pattern, asking questions of participants, responding to participant questions, etc.

2. Using grid paper with 10 x 10 grids, the instructor asked participants to fill in the squares to represent such decimal fractions as one hundredth, one tenth, thirty-three hundredths, seven tenths, etc. and mixed numbers such as two and one tenth. She used an overhead of the grid to help participants follow the process.

She would say something like, "Fill in three tenths and tell me how/why you did it that way." Someone would explain what they had done and why. Usually, especially in the beginning, they would try just to give the "answer"; and the instructor would prompt them again to explain what their thinking had been. The instructor then filled in the answer on the overhead, discussing related concepts as she went. Often participants would ask questions or offer alternate ways of thinking of the situation.

She also used the grid to introduce comparison of decimal fractions. She might direct attention to representations of 3-tenths and 3-hundredths and ask participants which was larger. Then she would have participants fill in representations of pairs of decimal fractions, (for example, .3 and .13) and decide which number was larger.

This work laid the groundwork for (later in the class) comparing decimal numbers without making representations of them. The instructor explained, for instance, that with .3 and .13, adding a zero to .3 (giving .30) makes both numbers show the same decimal parts and therefore makes it easier to compare them.

Once participants were somewhat comfortable with tenths and hundredths, the class talked about how to represent and compare thousandths.

3. The instructor used base ten blocks to demonstrate some of the decimal parts as well. First she used a 10 x 10 block as 1 whole (similar to the grids above). Then she used the thousand-piece cube as 1 whole and the identified tenths, hundredths, and thousandths of that cube.
4. For practice with place value and with reading and writing decimal numbers, participants wrote the decimal numbers asked for on the "*Decimals*" grid. The instructor later gave a blank grid and had participants write numbers she stated orally.
5. The instructor used "*Number-Line Decimals*" to help participants think about decimal parts as equal divisions. It also taught them to think about such things as what decimal division is between one-tenth and two-tenths. The class worked on some of these as a group, discussing concepts as they went.
6. The "*Decimal Sense--Place Value*" worksheet gave participants an opportunity to practice or think about a number of decimal concepts. It covers several different ideas and, for these groups of participants, required considerable monitoring and input from the instructor. Each of the place value questions in #5 could be interpreted in two ways and that was confusing. On the other hand, it also offered an opportunity for further discussion of decimal concepts. (For instance, the question "How many hundredths are in 0.75?" could be "75" (75 hundredths altogether) or "5" (a 5 in the hundredths place or 5 hundredths in addition to 7 tenths.) This required instructor interpretation.)

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7. "*Decimal Sense*" gave further practice visualizing decimal numbers on a number line. It also provided review of changing place values when multiplying and dividing by 10, 100, and 1000.
8. The instructor did some additional discussion and practice for comparing decimals on the board and/or overhead. Then they used "*Going in Circles*" for further individual practice.
9. "*Lineup with Decimals*" was a final review activity for comparing decimals.

Performance Assessment Methods and Results:

- "*Lineup with Decimals*" proved to be a bit overwhelming for some participants who were not secure enough with comparing decimals to sort through that many numbers. Therefore, as a transitional activity, the instructor asked the participants to work with the decimal numbers in any one box and arrange them in order smallest to largest. This worked better than sorting through all of the possible combinations each time. One class did almost all of the squares in this manner for homework. Then, after participants were more familiar with the problem and with comparing the numbers, participants worked together to look for those that lined up.

Comments:

- The instructor used a lot of worksheets for this section. In some cases, she did not expect every participant to do or complete each worksheet. The worksheets were a vehicle for discussion and also a way to practice with the numbers so participants could see, understand, and work with the concepts and operations.
- The class used the worksheets and activities in the approximate order given above, but the instructor exercised some flexibility depending on time available and on participants' level of understanding.
- For one participant who showed competence and understanding of much of the above earlier than all the rest of the class, the instructor pulled out several sheets from a book called Have Fun with Math. These review sheets had some very interestingly worded problems that practiced skills he should have. He enjoyed working with those problems and made good progress on them.

Activity IV: Working with Decimal Numbers: Estimation, Computation, and Word/Work Problems

1. Estimation with addition and subtraction of decimal numbers.
2. Computation (addition and subtraction) with decimal numbers
3. Estimation with multiplication and division of decimal numbers.
4. Computation (multiplication and division) with decimal number
5. Word/Work problems with decimal numbers
6. Recognizing reasonable answers to computations and word/work problems

Resources and Materials Needed:

- # "*Rules for Computing with Decimals(1): Addition*" Mary Mayfield, Instructor, Lane Community College, 1997.
- # "*Rules for Computing with Decimals (2): Subtraction.*" Mary Mayfield, 1997.
- # "*Placing the Point 1,*" Calculate! Problem Solving with Calculators by Terrence G. Coburn, Creative Publications, Mountain View, CA. 1988.
- "*Placing the Point 2,*" Calculate!
- # "*Rules for Computing with Decimals (3): Multiplication.*" Mary Mayfield, 1997.
- "*Decimal estimation 1,*" Calculate!
- # "*Hit the Target,*" Calculate!
- "*Estimation Quotients*" Calculate!
- "*Rules for Computing with Decimals (4): Division*". Mary Mayfield, 1997.
- # "*Decimal Estimation 2,*" Calculate!
- # "*Which Answer is Reasonable?*" (compiled from actual test questions and answers.) Mary Mayfield, 1997.
- # "*Be Reasonable*", Problem Solving in Mathematics, Lane County Mathematics Project, Sixth Grade book, published by Dale Seymour, 1983.
- Appropriate worksheets or pages from workbooks for practice of addition, subtraction, multiplication, and division--singly and mixed (The instructor used appropriate pages from Contemporary's Math GED book and from Contemporary's Decimals booklet as well as some worksheets, but there are many possibilities for appropriate practice.)
- # "*Decimal Review,*" Mary Mayfield, 1995.

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- # "Secret Message Using Decimals," Have Fun with Math, Louis Grant Brandes, J. Weston Walch, Portland, Maine, 1988.
- # "Deciding When to Add, to Subtract, to Multiply, or to Divide," Mary Mayfield, 1995.
- # "Beginning Word Problems with Decimals" Mary Mayfield, 1996.
- # "Word Problems with Decimals," Mary Mayfield, 1996.
- # "Decimal Practice with Calories" (to go with "How many calories do I burn if I . . . ?" an excerpt from an OEA CHOICE newsletter.) Mary Mayfield, 1996.

Description of Instruction:

1. Instructor wrote on board or overhead some additions and subtractions with decimals numbers problems; then she asked participants to decide what would be a reasonable answer to each. Participants discussed and practiced rounding to estimate answers. Instructor also wrote a few problems with answers and had participants decide whether or not the answers were reasonable.
2. To extend this process into actual computation with decimal numbers, the instructor used the worksheets "*Rules for Computing with Decimals (1): Addition*" and "*Rules for Computing with Decimals (2): Subtraction*." With the worksheet the class did the following:
 - a) First, individuals or pairs decided on a reasonable estimated answer to each math problem.
 - b) Second, they used the calculator to get an exact answer to each math problem.
 - c) Third, they used this information and what participants remembered from past computations with decimals to write out accurate computations for each math problem.
 - d) Fourth, through group discussion, and with instructor facilitation, they created the rules for addition and wrote them down step by step under "RULES." They had to think sequentially: what step goes first, etc. Sometimes participants had different ideas about the steps; then they would have to discuss what really comes first, what really comes next. Among other things, the instructor made sure to include all steps. Often, participants skipped mention of a step that seemed obvious to them--but that was not obvious to others. The instructor asked if anyone else had other steps or added additional steps herself.
 - e) Fifth, under "What to Watch For," participants noted parts of the process that were either most important or easiest to overlook. With addition, for instance, this might be "be sure to line up decimals," or "whole number is always written to left of decimal point."

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3. The instructor used the same procedure to develop rules for subtracting with decimal numbers.
4. Participants had an opportunity in class and as homework to practice addition and subtraction of decimal numbers.
5. After the class had done some computation, the instructor would begin class sessions by putting 2 and 3 decimal computation problems on the board (whatever the class had worked on most recently or whatever she thought needed most practice). Doing these few problems was a quick way for participants to keep these skills fresh in their minds. The class discussed the processes, the answers and any questions that came up.
6. Before doing multiplication of decimal numbers, the class did some estimation exercises. First, the instructor put some multiplication problems on the board and asked the class how they would arrive at reasonable answers. They discussed different ways of estimating the answer and what constituted a reasonable answer.

Note: In these and other similar exercises, the instructor reminded participants that not everyone uses the same process or arrives at the same reasonable answer. She emphasized that the most important outcome of this kind of work was to be familiar with decimal numbers and with the process. As a result, they would recognize reasonable--and unreasonable--answers.

7. The class then worked with "*Placing the Point 1*" and "*Placing the Point 2*." The instructor led the class through the problems in "*Placing the Point 1*", with questions and comments similar to the following:
 - How would you estimate a reasonable answer to 12×15 ? Maybe 10×15 , giving 150. If 150 is a reasonable answer, where would you put the decimal point in 1800 to show the exact answer?
 - How would you estimate a reasonable answer to 0.12×0.15 ? Does that seem difficult to do? OK. It's not necessary to do them in order. Let's try one that seems easier to understand? Which one should we try?
 - Maybe 1.2×1.5 ? How would you estimate a reasonable answer to that? Maybe 1×1 , giving 1 -- or 1×1.5 , giving 1.5. In either case, where would you put the decimal point in 1800 to show the exact answer?
 - Now how would you estimate 0.12×1.5 ? Maybe 0.12×1 , giving 0.12 (except you know it will be somewhat bigger). How would you place the decimal point in 1800 to show the exact answer?
 - Let's return to 0.12×0.15 . Can we use what we've already done to help answer this one? Remember that it's OK to use what you learn in some part of a problem to help you in another part. What do you think? If $0.12 \times 1.5 = 1.800$, will 0.12×0.15 be larger or smaller than 1.800? Where would we put the decimal point in 1800 to show this smaller answer? **Note:** It wasn't clear to many participants that they needed to add a zero to the left of the 1 to show this answer. This became an opportunity to talk again about place value.

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8. The class worked on "*Rules for Computing with Decimals (3): Multiplication*," using the same procedure used for "*Addition*" and "*Subtraction*." After devising the rules, participants had time to practice multiplication of decimal numbers.
9. "*Decimal estimation 1*," and "*Hit the Target*" were used for further practice making sense of multiplying with decimal numbers. On both of these worksheets, participants used calculators to check out their "guesstimates." To make sure participants understood what to do (especially since these were not standard worksheets for computation practice), the instructor led the class through the first problem (sometimes more) on each worksheet. Then, they worked for several minutes in class (sometimes with a partner, particularly on "*Hit the Target*") so she could monitor for any difficulties. What participants had not finished in class became homework.
10. The class approached learning division of decimal numbers the same way. First, the instructor led the class through some discussion and practice of estimation of decimal divisions. Next, the class did the worksheet "*Estimating Quotients*" for more estimation practice. Then, they did "*Rules for Computing with Decimals (4): Division*". The instructor included lots of division practice because many participants had trouble doing division. And she made sure the practice problems included all possible permutations of decimal division: dividing by whole numbers, by mixed numbers and by decimal fractions(as divisors); having whole numbers, mixed numbers, and decimal fractions to be divided(as dividends). And, to improve understanding of and flexibility with decimal division participants did "*Decimal Estimation 2*."
11. The instructor used a number of worksheets as review of skills learned so far. These included "*Which Answer is Reasonable?*", "*Be Reasonable*," "*Decimal Review*," "*Secret Message Using Decimals*" and appropriate mixed computation worksheets or workbook pages. When participants were doing computation review (rather than estimation or reasonable answer review), the instructor suggested that they keep their "*Rules for Computing with Decimals: (1-4)*" sheets handy for reference when needed.
12. To begin working on word problems, the instructor handed out "*Deciding When to Add, to Subtract, to Multiply, or to Divide*." Even though, the examples on this hand-out don't really involve decimal numbers, the instructor sometimes had participants write (but not solve) the computation they needed to do for each example. Then, she did some decimal word problem examples on the overhead. Instructor handed out "*Beginning Word Problems*" and asked participants only to figure out whether they would add, subtract, multiply or divide to find the solution to the problem--and write A, S, M, or D next to the problem to indicate their decision. Then they were to write down the computation they would use to solve the problem, but not do the actual calculating. She wanted them to focus only on the strategy for solving the problems.

Note: Participants had trouble here. Consequently, the instructor worked with them in the beginning to set the pace. With practice, the process became easier and more accurate.

- First, they wanted to get "the answer," and so found it difficult to just think about what operation to use, etc.
 - Second, they had trouble choosing an appropriate operation (or sequence of operations) and/or setting up the calculation correctly (a common problem was setting up a division backwards).
13. Finally, when participants were comfortable choosing operations and setting up calculations, the instructor asked them to calculate answers for the problems they had been working on. Then they continued solving the rest of the problems on this worksheet..
14. Participants worked on "*Word Problems with Decimals*" and "*Decimal Practice with Calories*." Periodically, the instructor led a discussion of the problems they had done. Questions focused on which operation a participant selected, and the decision process for using that operation. Sometimes the instructor asked "If you were to refer to the hand-out, how would you know which operation to use here?" This was also an opportunity for participants to check the answers of the work they had done so far.

Comments:

- Because it was easier for participants to estimate addition and subtraction than multiplication and division, the class spent considerably more time estimating with multiplication and division than with addition and subtraction.
- Participants found it difficult to understand the concept of multiplying as finding a fractional (or decimal) part of another quantity. They learned in elementary school that multiplying numbers produces a larger number; so when an answer comes out smaller than one of the numbers they started with, it confused them. The class worked with this concept enough that they accepted it, but some participants still had trouble believing that this could be a reasonable answer.
- The class was working on estimating computations, participants often tried to use their (usually foggy) memory of computation rules, rather than their understanding of decimal numbers and the computation processes. The instructor emphasized that the class would work with rules for computation, but that, for the moment, it was important to develop understanding of these processes.
- The structure "*Rules for Computing with Decimals: (1-4)*" was designed to first help people develop and think about their common sense understanding of decimal computation. Then, with this understanding and an exact calculator answer, they could, hopefully, make some connection between common sense and the rules. Also, by having their estimation, exact answer and process at the top of their written rules, they could use this and the rules for review.

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Two participants did not remember enough about decimal computation to use their estimations and the calculator answers to calculate the sample problems and recreate the computation rules. In that case, the instructor gave them one or two main rules (as, for addition and subtraction, to line up the decimal points) and they worked from there. She gave additional support as necessary.

Note: The instructor expected some participants to protest having to write out the rules, but not one person in any of the classes ever complained about doing it.

- In choosing the problems for "*Rules for Computing with Decimals: (1-4)*," the instructor tried to touch on most, if not all, possible complications that might be involved. Therefore, problems chosen included varying numbers of decimal places, whole numbers written with no decimal point, problems with mixed numbers, problems with just decimal fractions, division by whole numbers and by decimal fractions, etc.
- The "hidden message" in working with "*Decimal Estimation 1*," "*Decimal Estimation 2*," and "*Hit the Target*" was that often it is OK not to get the right answer on your first try. Sometimes you can try something, see how close you get, and use the information to try for a better approximation. A second "message" was that it helps to have (or develop) a plan, some sort of organized approach to the problem.
- The instructor used many worksheets for estimating with multiplication and division. The goal was for participants to practice as much as possible with thinking through decimal multiplication and division in different contexts. Once they thoroughly understand the computation processes, they are able to recognize reasonable (or unreasonable) answers in their work. How much each participant did depended on skill level, how quickly they worked, and how much time they had to work on math at home.
- "*Hit the Target*" seemed to be the most difficult of the exercises for most participants. That is why the instructor placed it at the end of the multiplication sequence above.
- The instructor first used "*Be Reasonable*" as an early activity in teaching division of decimal numbers. It was difficult for most participants to switch back and forth from multiplication to division. Also, there were too many types of questions for their level of comfort. It worked better as a review option.
- Note that "*Decimal Review*" is a matching exercise and best presented on two separate sheets of paper rather than back to back.
- "*Secret Message Using Decimals*" has several problems that require understanding some fraction-decimal conversion. It is most appropriate for review after fraction-decimal conversion has been learned.
- "*Deciding When to Add, to Subtract, to Multiply, or to Divide*" could be used in working with whole number problems. It would be helpful if a class had trouble understanding word problems involving whole numbers. For these classes, the instructor anticipated significant confusion in working with word problems involving decimals, so she introduced the hand-out there.

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- Problems in "*Word Problems with Decimals*" are significantly more difficult than those in "*Beginning Word Problems with Decimals*." Participants particularly had difficulty with #7 and #8 of "*Word Problems with Decimals*."

Activity V: Fraction-Decimal Conversion

1. Converting decimals to fractions
2. Converting fractions to decimals
3. Using the fraction keys on a scientific calculator to do fraction-decimal conversions

Resources and Materials Needed:

- Appropriate practice set of decimals to convert to fractions
- Appropriate practice set of fractions to convert to decimals
- Basic 4-function calculator
- Scientific calculator with fraction keys
- # "Decimal-Fraction Conversions" David Shellabarger, Instructor, Lane Community College, 1995.
- # "Equal Fractions and Decimals" Have Fun with Math, Louis Grant Brandes, 1988.

Description of Instruction:

1. The instructor wrote some decimal numbers on the board and led a review discussion of place value and naming decimals. She then asked the class how they would write some of the decimals as fractions. It was a pretty easy step to, for example, $.3 = 3/10$ or $.025 = 25/1000$. Participants did, however, need some instructional review in simplifying the fractions, which the instructor explained and demonstrated. Participants then practiced with some examples.
2. The instructor wrote some fractions on the board and discussed with the class the meaning of fractions. For example, three-fourths can mean 3 parts out of 4, a ratio of 3 to 4, or 3 items divided into 4 groups. She explained that the standard method of converting fractions to decimals is based on this last meaning. Three-fourths becomes 3 divided by 4, (which, by dividing using decimal division skills or a calculator) becomes $.75$. She reminded participants that always the numerator must be divided by the denominator.

She demonstrated some conversions (such as $2/3$) that don't divide evenly. She discussed how these would be rounded to the appropriate decimal places.

Participants then practiced converting some fractions to decimals, both by hand and using a basic 4-function calculator.

3. Later, after participants learned using proportions, the instructor explained that if they knew what accuracy (place value) they needed their decimal answer to be, they could set a proportion. For example $3/4 = ?/100$, results in $75/100$, which they can convert to as a decimal. For a few participants who had trouble remembering to divide or who often got their division "backwards," this seemed easier to work with.

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4. The instructor brought in some scientific calculators with fraction keys. She explained and demonstrated how to enter fractional numbers on these calculators. She pointed out how those numbers appear on the display. Also, she taught participants how to do the fraction-decimal conversions and fraction computations on these calculators. Participants had some chance to try out all these operations on the calculators and to ask other questions about the calculators.
5. "*Decimal-Fraction Conversions*" and "*Equal Fractions and Decimals*" were used to wrap up this section. Participants did some in class and the rest for homework. Often participants worked in pairs on "*Equal Fractions and Decimals*."

Performance Assessment Methods and Results:

- The instructor monitored what participants did in class and gave feedback as needed.
- "*Decimal-Fraction Conversions*" and "*Equal Fractions and Decimals*" gave a good indication of participants' level of competence and comfort.

Comments:

- This company uses few fractions in their work, except for a little measurement and a little fraction-decimal conversion. This class included only the needed work with fractions. Also, time was limited and many of the other topic areas could have used more time for practice, so there was little temptation to include more about fractions.
- Participants did not show much interest in using a consistent method for rounding numbers. Also, most of the conversions they do are with tape measure fractions that come out even. The difference in whether they round up or just drop digits past the place value they need would generally not be significant in their work. So, with lots more relevant topics to cover, the instructor did not spend much time on rounding.
- The time spent working with the scientific calculator with fraction keys was quite short, but participants found it useful and interesting. A major goal of this part was to help them feel more familiar with and less intimidated by the technology. The instructor wanted them to have some awareness of the what the potential of that technology is. To that end, they spent a brief time discussing other features of that calculator and even discussed briefly financial calculators and graphing calculators. Some participants found these calculators so useful that they bought their own.
- "*Decimal-Fraction Conversions*" was a fairly straightforward worksheet; participants could just apply the rules. "*Equal Fractions and Decimals*" required quite a bit more processing to sort through the options to find the ones that were equal. Many participants had trouble with this and needed extra support from the instructor. The instructor used this partly as an opportunity to stress thinking about reasonable answers. For instance, "Is it reasonable that 8.75 could equal either $\frac{7}{8}$ or .875 or $\frac{3}{4}$? If it is not reasonable, then we can eliminate 8.75 from our thinking and look more closely at the other three."

Activity VI: Working with Ratios and Proportions

1. Understand ratios
2. Understand and solve proportions
3. Set up proportions to solve word/work problems

Resources and Materials Needed:

- "Unit 4: Ratio and Proportion," pages 128-149, Decimals, Fractions, Ratios, and Percents (part of the series called SMART SOLUTIONS: Skills, Problem Solving, Tools, and Applications), 1995, Ellen Carley Frechette, New Readers Press, Publishing Division of Laubach Literacy International, Box 131, Syracuse, New York 13210-0131
- # "Problems Stated as Ratios (Teacher Tips page and worksheet page)," GED Priority Math, pp. 4-5, Mary Jane Schmitt, Massachusetts Department of Education, 1989.
- # "General Multiplication/Division Word Problems (Teacher Tips page and worksheet page)," pp. 6-7, GED Priority Math.
- Ratio and Proportion, Connie Eichhorn, Cambridge Adult Education, Power Math Series, pp. 53 and 56.
- # "Similar Shapes (Teacher Tips page and worksheet page)," pp. 12-13, GED Priority Math.
- # "Using Proportions: Estimation of Fish Population," copied from old discarded elementary math books.
- # "Using Proportions: Bicycle Gears," copied from old discarded elementary math books.
- # "Using Proportions: Model Airplanes" copied from old discarded elementary math books.
- # "Practice with Ratios and Proportions," Mary Mayfield, Instructor, Lane Community College, 1996.
- # "Direct Variation," Mary Mayfield, 1996.
- # "'Per' Problems," Mary Mayfield, 1997.

Description of Instruction:

1. The "Ratio and Proportion" unit of Decimals, Fractions, Ratios, and Percents does a good (and fairly complete) presentation of ratios and proportions that matched pretty closely the skill level of participants in these classes. It covers virtually all the concepts the instructor considered important. Therefore, it made a good base text for this section. The rest of the activities for this unit are supplemental to that text.

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2. The instructor talked about ratios and gave examples of ratios in the room:

- the ratio of people wearing hats to people not wearing hats
- the ratios of chairs to tables
- the ratio of people under 25 to all people; etc.

After working through approximately pages 128-135 in the text, "*Problems Stated as Ratios*" was used to give participants additional, somewhat more sophisticated practice with ratios. The instructor gave participants the "Teacher Tips" page as well as the worksheet because she thought that the "Teacher Tips" page showed clearly how to write and simplify ratios. Participants looked over those examples and discussed any that seemed confusing. Then they worked the problems on the worksheet. The instructor monitored their work and gave feedback as needed.

3. The instructor preferred to work with unit rates (and what she refers to as "per" problems) after the class had learned to work with proportions, so she skipped pages 136-137 of "*Unit 4: Ratio and Proportion*" until later. See #8 below.
4. The instructor introduced the idea of equal ratios (or proportions), by choosing examples in the room of ratios that could be stated in more than one way (by thinking of the objects as grouped in some way). She also used blocks to demonstrate the concept.
5. The next step was to show a proportion with one piece of information (one number) missing and ask participants how they might find that number.
- For the first example or two, the instructor tried to choose a problem like $7/9 = 105/?$ or $6/9 = ?/15$ that wouldn't be obvious but would come out even.
 - Then she included one that didn't come out even.
 - Then participants practiced, using the problems on page 139 of the text.
6. In introducing solving word/work problems using proportions, the instructor emphasized that one key to setting up the proportion correctly is that the labeling must be parallel in each ratio. For instance, if the first ratio is oranges/cents, the second ratio must also be oranges/cents. She suggested that continuing to use a chart or table to set up their problems or being careful to label the amounts as they set up the proportions would help keep this clear.

Good supplements to the text for introducing work with solving word/work problems with proportions were: The "Teacher Tips" page on "*General Multiplication/Division Word Problems*" and the two practice pages (53 and 56) from *Ratio and Proportion*. The "Teacher Tips" page gave clear examples, working through the problems a bit differently than the text. The practice pages gave lots of straightforward, basic practice for those who needed it.

Note: The instructor used the following additional worksheets to give participants further practice with the process, but especially to familiarize them with the wide variety of situations in which understanding and using proportions can be useful: "*Similar Shapes* (Teacher Tips page and worksheet page), "*Using Proportions: Estimation of Fish Population*," "*Using Proportions:*

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Bicycle Gears," "Using Proportions: Model Airplanes," "Practice with Ratios and Proportions," "Direct Variation," "Per Problems."

7. When participants seemed comfortable using proportions, the instructor introduced "Per" problems or unit rates. She created this special unit because she had seen participants struggle with this type of problem in the pre-test and other situations. Even when the problem clearly stated something like "safety cost is figured as cost per hour worked, that is, the total cost of safety incidents divided by the total hours worked," participants would either do the division backwards or be confused in other ways. She created it because she felt this was not only an important concept for some of them in their work, but also that it was a concept prevalent in many areas of life.
 - To introduce this unit, the instructor first talked briefly about unit rates and unit pricing. She asked the participants to name examples (miles per gallon, cost per hour, etc.).
 - Then she divided the class into three groups and had each group study one way of approaching these problems. (For example, setting a ratio and reducing it--as in "*Unit 4: Ratio and Proportion*", page 136-137; using divisions--as on the worksheet, example 1; or using a proportion--as on the worksheet, example 2).
 - They were to read the examples for their approach, try some other problems using that approach, try explaining the approach to other members of their group, ask questions of the instructor, etc.
 - Then each group, with the instructor's help as needed, explained their approach to the whole class. See Comments below.

Performance Assessment Methods and Results:

- The instructor closely monitored participants as they worked and gave assistance and feedback as needed.

Comments:

- The instructor sees proportions as one of the most useful and practical of basic math tools. She allowed a lot of time and practice for participants to become familiar both with the types of situations where proportions could be useful and with actually using the proportions to solve problems.
- Some participants found the solving proportions technique as shown in the book confusing or complicated. They had trouble following through the steps. (At least one participant, on the other hand, liked the technique so well, she wrote out each step every time she solved a proportion; she felt it kept her organized.)

The instructor went through the steps and reasoning of this process several times. She connected it with the idea of cross-products being equal in a proportion. She also pointed out that the technique could be summarized as "Multiply to get the cross-product of the 2 numbers you have diagonal to each other; then divide by the third number."
- Explaining the process for solving "per" problems (see #8 above) was challenging for most participants. Both the instructor and the company put high priority on participants

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understanding procedures and their thought processes, and being able to explain them to others. Some participants "explained" by giving a list of computations, as: "multiply 400 by 1, then divide by 8." The instructor helped participants both to talk about the process or their thinking steps, and to generalize the instructions.

- A number of participants brought in examples from home or work where they were able to use a proportion to solve a problem. Some examples included finding the weight of 100 feet of pipe when the weight of a certain length was known or mixing various quantities of spray when you know the standard mixing ratio. Another example about increasing the size of an anchor a participant was making, led to a discussion of proportional volume versus proportional length or width.
- There were a number of places in the company where understanding proportions could be useful. Some of these show up here, some in the later review worksheets, and some in the algebra/geometry curriculum.

Activity VII: Working with Percents

1. Develop understanding of percent as per hundred
2. Learn to set up and solve percent problems

Resources and Materials Needed:

- Base ten blocks
- # "Focus Student Activity 39.1," Visual Mathematics, Course 1, "Modeling Percent," The Math Learning Center, Salem, OR, 1994.
- # "Focus Student Activity 39.2," Visual Mathematics, Course 1
- # "Percent means Per Hundred" Mary Mayfield, Instructor, Lane Community College, 1996.
- # (10 x 10 grid paper) # see Activity III, Basic Math Curriculum.
- "Which of These Is .5%?," Mary Mayfield compiled from participant representations of .5% on pre-test, 1996.
- # "Activity Sheet VII-1-A," Math and the Mind's Eye, The Math Learning Center, Salem, OR, 1991.
- "Activity Sheet VII-1-B," Math and the Mind's Eye
- # "Activity Sheet VII-1-C," Math and the Mind's Eye
- # "Activity Sheet VII-1-D," Math and the Mind's Eye
- "Equal Fractions, Decimals, and Percents," Have Fun with Math, J. Weston Walch, Publisher, Portland, Maine, 1988.
- # "Quick and Easy Everyday Percents," Mary Mayfield, 1996.
- # "Solving Percent Problems" Mary Mayfield, 1978, (revised 1997)
- # "Percentage" ("Teacher Tips" page and worksheet), "Ratio and Proportion", GED Priority Math, Mary Ann Schmidt, Massachusetts Department of Education, 1989.
- # "Percent Problem Chart," Have Fun with Math
- Figures from Math and the Mind's Eye, Unit VII, Activity 1 ("Introduction to Percentages") and Activity 2 ("Fractions, Decimals & Percents")
- "Lineup with Fractions, Decimals, and Percents," Have Fun with Math, J. Weston Walch, Publisher, Portland, Maine, 1988.

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- # "Percent Computation Review," Mary Mayfield, 1997.
- # "Mystic Square with Percent," Have Fun with Math, Louis Grant Brandes, J. Weston Walch, Portland, Maine, 1988.
- "The Percent Equation" (pp. 164, 165) Decimals, Fractions, Ratios, and Percents, Ellen Carley Frechette (part of the Smart Solutions series), New Readers Press, Syracuse, NY, 1995.
- "Solving a Percent Equation" (pp. 166,167), Decimals, Fractions, Ratios, and Percents

Description of Instruction:

1. Both Math and the Mind's Eye and Visual Mathematics have teacher materials and worksheets that are very helpful in introducing percents and working with the concept of percents. The instructor used many of the ideas and activities from these materials.
2. The instructor's "chant" for this section was some version of "Percent means hundredths" or "Percent means how many out of 100." She began illustrating this with 100 (10 x 10) base 10 blocks, showing whole number percents (7%, 15%), asking participants how they would represent 50%, 99%, etc. There was some review of decimals, especially hundredths, having participants remember how to write 7 hundredths, 15 hundredths, etc. with decimals and explaining that, for example, 7% is another way of writing (.07).
3. The instructor asked participants to think about, then show and/or explain how they might illustrate $\frac{1}{2}\%$ or 200%. There was some discussion about how you could have 200 out of 100--or even how you could have anything over 100%. The instructor introduced the vocabulary of 100 as a base or a standard. They used the word "standard" in their job, so they had heard or maybe used such terminology as "101% of standard." The instructor also gave such examples as: if you had a herd of 100 cattle and each of them had one calf, you would then have 200 head of cattle, or 200% of your original herd.
4. The instructor introduced the 10 x 10 grid paper and used an overhead of those grids to show some percents. Then she gave participants "Focus Student Activity 39.1" and "Focus Student Activity 39.2." They worked first on (...39.1) with the instructor monitoring and answering questions; then they worked some on (...39.2), again with the instructor monitoring and answering questions. Whatever didn't get finished at that time was assigned as homework.
5. The class discussed these activities (and any difficulties or confusion participants had had) at the beginning of the next class, beginning with (...39.1).
 - The instructor encouraged participants to explain the thinking they went through to get their answers.
 - The instructor facilitated discussion so that all approaches were shared, that differences in answers were accounted for by process, and that there was some understanding of the most accurate or most efficient ways of obtaining the answer.
 - For instance, on (...39.1), (i) was seen as shading half of a 5-square rectangle.

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- (m) was seen in several ways, the most efficient one being the shaded section being a replica of the un-shaded section.
- For (k) participants were tempted to call it 25% with no clear reasoning, or to estimate the parts of squares and get a rough idea, but the most accurate and efficient way seemed to be to create 4 rectangles, each $(3 \text{ squares}) \times (4 \text{ squares})$ and each with half of the rectangle shaded. This created 4 sets of 6 shaded squares plus one square in the middle that wasn't part of any of the rectangles.

Reviewing (...39.2) went more quickly. There was some confusion with fractional percents and .75% was confused with 75%, but participants were able to interact with each other to clear up most of these confusions.

6. The instructor gave participants "*Percent Means Per Hundred*" as a reference and a reminder of the different possible representations of hundredths and of the basic concepts of percent.
7. The instructor then handed out "*Which of These is .5%?*". To create this worksheet, the instructor used the 10 x 10 grid to reproduce the representations participants had made of .5% on their pre-test of .5%. On this worksheet, she asked participants to find a representation of .5%, to explain how they knew it was .5%, and to name what percent was represented by some of the other illustrations. There was some discussion of what someone might have been thinking when they drew the incorrect representations and some discussion that some of the illustrations really were difficult/impossible to interpret as a percent.
8. The instructor used the Math and the Mind's Eye Activity Sheets as further practice with thinking in terms of percents. Not every participant completed every worksheet, but in most classes, the instructor used an overhead to demonstrate and discuss the concepts and techniques. She helped individual participants work with enough of the problems to get a feel for the concept.
9. The instructor worked with participants to model some percent situations (using base 10 blocks), such as: "This year tickets to the concert cost \$15, which is 125% of the cost last year," or "The students need \$2000 for a class trip. They have \$880 so far." This was quite challenging for virtually all participants, so the instructor did most of the modeling and explanation with input from participants as they could.
10. "*Quick and Easy Everyday Percents*" was designed to help participants recognize and work mentally with common percents. The idea of the worksheet was that if you know how to work with a few basic percents, you can use these in combination to mentally compute (or at least estimate) many percent problems. The instructor worked through the first several lines of this with participants, using overhead or white board. It was confusing for some participants, but helpful for others.
Note: It is important to work across the sheet from left to right rather than try to work down the columns.
11. To introduce using proportions to solve percent problems the instructor used "*Solving Percent Problems*" and "*Percentage*." She worked through several problems on overhead or white board using the steps shown in the left column of "*Solving Percent*"

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Problems." Then she had participants work through the examples in the right column. Finally, she offered "*Percentage*" (*Teacher Tips page*) as another way of showing this same procedure.

12. For those participants who remembered (more or less clearly) doing percent problems by multiplying or dividing, the instructor suggested that they first write out the problem in the form: $\underline{\hspace{1cm}}\%$ of $\underline{\hspace{1cm}}$ is $\underline{\hspace{1cm}}$. Then she suggested that they convert the percent (if they knew it) to a decimal and translate what they had written to a math statement:

$\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$. Then, if they have the two numbers to multiply, they should multiply. If not, they should divide the number they have on the right of the equal sign by the number they have on the left of the equal sign. Participants who wanted to use this method of figuring percents used pages 164-167 from Decimals, Fractions, Ratios, and Proportions for instruction and practice.

13. Participants used the "*Percentage*" worksheet to start setting up percent problems. The instructor mostly monitored and answered questions as participants did these, but occasionally she would set a problem up on the board or overhead, if it seemed necessary. "*Percent Problem Chart*" and other similar problems sets were used for further practice in class and/or as homework.
14. The instructor used the figures from Activities 1 and 2 of Math and the Mind's Eye, Unit VII as a warm-up, after the class had some understanding of naming percents and of the connection between percents and proportions. The instructor asked whether figure A or figure B has a larger portion of its area shaded, and participants had to explain their reasoning. This one seemed quite clear and easy. Then the same question was asked about figures C and D. This one is less clear. Again, participants explained their reasoning. Even if they weren't sure of an answer to the question, sometimes they were asked for their thinking. Someone might suggest putting four figure C's together to see how many shaded squares there would be out of 100 squares. If no one suggested using a proportion, the instructor would mention this possibility--and would show that either $9/25 = ?/100$ or $?/25 = 39/100$ would give the information needed to compare the two figures.

For the two figures from Activity 2, the instructor just asked what percent of the figure was shaded. She might ask them to draw a way of figuring it out, as well as using a proportion, or any other options.

15. The instructor used "*Lineup with Fractions, Decimals, and Percents*" a couple of different ways for different participants or at different times. It (or part of it) was first used as a practice sheet for changing fractions and decimals to percents. The instructor suggested that, especially as they were so familiar with proportions, using a proportion was probably the easiest way to do that.
- For instance, to change $3/4$ to percent, they would use $3/4 = ?/100$.
 - To change .8 to a percent, they might use $8/10 = ?/100$.
 - The instructor emphasized that some of the numbers might be so familiar that they just know the percent equivalent.

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- Some of the decimals might already be hundredths and therefore be easy to change to percent, as $.62 = 62\%$. They discussed the fact that hundredths is 2 decimal places and that one way to change a decimal to a percent is to move the decimal places. (Participants were free to choose their approach to the problem; and the instructor offered support as needed.)

"Lineup with Fractions, Decimals, and Percents" (or part of it) was also used with the given directions--for participants to find a "lineup" of numbers. This was particularly appropriate for participants with good understanding of fractions, decimals, and percents. And was also a good follow-up for the previous activity of changing fractions and decimals to percents.

16. The class also spent some time talking about percent of increase and percent of decrease--also percent of overweight and underweight (concepts used often in the plant). The instructor did not have materials that she liked to use here, so most teaching was done in discussions and on the board or overhead. She had them think about problems such as:

- 500 increased (or decreased) by 10%
- What is the percent of increase when cost rises from \$8 to \$10
- What is the percent of decrease when cost drops from \$10 to \$8

They reviewed the concept of base or standards, because percent of increase or decrease is based on whatever the original cost (or other number) was. That is, the original is used as the base or standard. The instructor wrote a few problems on board or overhead for practice.

17. *"Percent Computation Review"* and *"Mystic Square with Percent"* were used for review. For *"Mystic Square with Percent,"* participants worked in pairs. They needed to agree on an answer and one or more processes for finding that answer. Also, they needed to be able to explain their process. Participants did *"Percent Computation Review"* individually--partly in class with instructor monitoring and partly as homework.

Performance Assessment Methods and Results:

- Review sheets were designed for assessment. As participants worked together on *"Mystic Square with Percent,"* the instructor could assess their competence with concepts and skills and also their ability to problem-solve, draw conclusions, and communicate with another person about what they were doing..
- Participant competence with proportions carried over to this section. Everyone learned (most rather easily) to set up percent problems using proportions, and most participants chose this as their standard approach to percent problems.

Comments:

- This instructor prefers to emphasize solving percent problems using proportions because:
 - a) It is her experience that people find this method less confusing and easier to remember than the more standard approaches to solving percents.

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- b) It means that people can concentrate on getting good at one technique (using proportions), thereby acquiring the skill to solve many, many different kinds of problems.

However, she believes that the best approach is a flexible one. If participants remember a different method of percents well enough that they want to continue to use it, she will help them "brush up" and practice that. Or, if using proportions to solve percent problems doesn't seem to be working well for someone, she'll help them find a method that works better for them.

- Note that both "*Solving Percent Problems*" and "*Percentages*" set up the proportion using the following reasoning:

percent : 100 = part : whole or base

Some participants used the reasoning: part : percent = whole or base : 100

If they could explain their reasoning and use it consistently (or could consistently use one of these two types of reasoning to always set up a correct proportion), the instructor supported their approach.

- Sometimes the instructor would give only the "*Solving Percent Problems*" handout to illustrate using proportions to solve percent problems. At other times, she would hand out both that and "*Percentage*" so that participants could use for reference whichever one seemed clearer to them.
- The Math and The Mind's Eye curriculum materials are wonderful. They are diverse, geared toward active learning and problem-solving, and give a lot of instructional support.

Activity VIII: Practical Applications On the Job and Off

1. Give participants practice with common sense and problem-solving approaches in a word/work problem context.
2. Use a variety of word problems to practice math skills.
3. Use a variety of work-related problems to provide an opportunity for participants to become more familiar with the work context (over-weighs, safety costs, etc.).
4. Help participants become more competent and confident in applying their math skills to work situations

Resources and Materials Needed:

- # "*Deciding When to Add, to Subtract, to Multiply, or to Divide*," # (see Activity IV Basic Math Curriculum) Mary Mayfield, Instructor, Lane Community College, 1995.
- "*Problem Solving Possibilities*," Mary Mayfield, 1996.
- ## "*Concentrate on What You Know*," Mary Mayfield, 1996. ## (This worksheet can be found in the Flyer Reading curriculum.)
- "*Focus Master A (Situations)*" "*Modeling Percent*," *Visual Mathematics*, Course 1, The Math Learning Center, Salem, OR, 1994.
- "*Word Problems with Percents and Proportions*", Mary Mayfield, 1996.
- # "*Work-Related Math A, B, C, D, E*," (B not included) Mary Mayfield, 1996-97.
- Various workplace documents

- This mark indicates a copy of the handout is in the **Handout** section of a different curriculum indicated in the text.

Description of Instruction:

1. The instructor made sure that each participant still had a copy of "*Deciding When to Add, to Subtract, to Multiply, and to Divide*" in case they needed it for reference.
2. The instructor handed out "*Problem Solving Possibilities*." This was an opportunity to think about and discuss how we approach problem solving. This hand-out encourages an organized approach to problem-solving:
 - Understand the problem
 - Plan your approach
 - Carry out your plan
 - Look back to see that your answer works.

The class discussed and practiced this problem-solving plan and some of the suggested problem-solving strategies. They talked some about how to apply it in a work setting.

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3. "*Concentrate on What You Know*" was designed to be a "light" homework assignment. It focused discussion and thinking on finding the information needed to solve a problem without getting waylaid by insignificant information.
4. The instructor had participants set up percent statements using the information given in "*Situations*." Most participants used proportions to do this, although they were free to use a percent equation if they preferred. Sometimes the instructor would use base 10 blocks, drawings, or whatever was convenient and appropriate to model the situation if participants were having trouble setting up a statement.
5. The instructor chose varied, straightforward, and realistic questions for "*Word Problems with Percents and Proportions*." She had the class start out slowly on these, deciding first how to set the problem up. After giving them some time to set up a few problems, she led a discussion about how they did that. She pointed out instances when more than one method worked as well as when a particular method would not work. Participants then finished the sheet. They worked some in class with instructor monitoring and answering questions and did the rest as homework.
6. The instructor collected a lot of math examples related to work at this pvc manufacturing company. She compiled these into "*Work-Related Math A, B, C, D, and E*" and used them as math practice, and as an opportunity to discuss and better understand the various work concepts and processes. The last section on "*Work-Related Math D*" (#7-12) was designed to remind participants to pay careful attention to detail. It demonstrated that they could often figure the same statistic, etc. from different information.

"*Work-Related Math E*" was designed to walk participants through the standard math steps to find percent of overweight or percent of underweight. Once they knew that, then they compared it to the formula (in #9) that they use in the plant to figure these.
7. The instructor also used various actual workplace documents in the class. Some of these just focused discussion about certain concepts or processes. Most were used to think about or practice the math skills involved.
 - For example, the instructor copied a quarterly bonus plan calculation report (which showed 3 months of information on conversions costs, material utilization, safety costs, QA rejects, and Regrind) without the final monthly or quarterly percents.
 - Then she asked participants to explain the concepts and to figure the monthly and quarterly percents.

Performance Assessment Methods and Results:

- When participants worked on these problems individually, the instructor monitored their work carefully both to see how their skills were progressing and to offer hints or support as immediately as possible to reinforce their skills.

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Comments:

- Many participants still didn't consistently look back to see that their answers worked . Over time, though, people began to realize how often they could catch a significant mistake just by checking their answers for reasonableness.
- By this time, participants would sometimes bring in examples from their work that they didn't understand. Often these made interesting and useful class discussion problems.

Activity IX: Concluding Activities

1. Complete grant paperwork
2. Give Math Training Post test
3. Provide feedback to participants, and get feedback from participants on class
4. Complete Individualized Learning Plans (Part 1 was completed at the beginning of class)

Resources and Materials Needed:

- "Learner Assessment Form"
- "Individualized Learning Plan"
- "Learner Evaluation Form"
- # "Math Review Post test" Mary Mayfield, Instructor, Lane Community College, 1995.
- # "Math Competencies" Mary Mayfield, 1997.

Description of Instruction:

1. The class completes the paperwork for the grant.
2. Each participant completed their "Individualized Learning Plan."
3. The class takes the post test.
4. The instructor completed a "Math Competencies" sheet for each participant, showing the level she thought they had attained for each competency and making comments for clarification as appropriate.
5. Each participant had an opportunity to talk with the instructor regarding the competencies, their progress in the class, etc.

Performance Assessment Methods and Results:

- Participants were given the opportunity to review the progress between the pre- and post test results.
- The instructor evaluated each participant's overall performance and their skills as shown on the post test. She summarized these for each participant on the "Math Competencies" sheet.
- Participants also had a chance to provide input to the instructor on the quality of the class.

Course Summary

Background:

In the Job Task report, focus groups identified the need for math trainings at both basic and advanced levels. Employees in all departments keep records and perform calculations using fractions, percentages, and formulas. Although the term "math anxiety" was not used, some participants in focus groups felt that employees lacked self-confidence in math and were uncomfortable working with numbers.

Expectations:

Company:

The company wanted to devise minimum level math competencies for production workers. The math instructor worked with Site Advisory Team members to devise a math screening test that assessed these minimum math competencies required by the job. The screening assessment developed specifically for this worksite was mandatory for all production workers, although math training was voluntary.

Learner:

The instructor reviewed individual scores with each person who took the test. She helped individuals interpret the scores and discussed with them their interest in improving math skills. For those who did not need this basic math class, the instructor made note of needs and interests to address in future classes. The participants who voluntarily joined the basic math class needed to upgrade basic math skills.

Instructor:

By working with the Site Advisory Team to design the math assessment, the instructor obtained a good picture of what the company expected its workers to know and some of the ways the company expected production workers to use that information. From the results of the assessment and conversations with employees who took the test, the instructor learned that class participants would have a wide range of skills and that some participants would be reluctant to participate.

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Evaluations:

Company:

The Site Advisory Team originally planned the course for twelve weeks. When the instructor expressed concern that participants needed more time to reach competencies, the team agreed to extend the class another five weeks. Although the company offered participants the option to stop the training after the initial twelve weeks, only two employees chose to do so.

Learner:

Although initially some participants may have had resistance to taking the class, many commented on how much they learned. Class participants worked together well and over time became quite interactive. They became comfortable asking questions, taking risks, and suggesting changes in the class that would better fit their needs. Everyone who completed the 17 week session developed a sense of mastery of useful math skills. In addition, most participants developed a greater comfort level in working through problems together.

Instructor:

The instructor commented that participants, though sometimes hesitant, showed willingness in trying new ways of doing things. Through continuous monitoring of progress over the length of the class, the instructor reinforced participants newly learned skills.

Handouts

Math Training Preview

Here are a few questions that will help me decide how to plan this class--by knowing a bit more about your expectations and your math skills and experiences. If you have trouble getting it down in writing, check with me and I'll try to help.

1. Why are you taking this class?

2. What would you like to know more about or be able to do better when you finish this class?

3. What math classes have you taken? When? How much of that math would you say is still retained in your memory banks? (most, some, none)

4. What was math like for you in school--elementary, junior high, high school, college, whatever? (Anything significant for you would be helpful to me here. For instance: what math did you like or feel you were good at? what was your worst math experience? what were your math teachers like? Etc.

5. In what ways might knowing more math help you on the job? Can you think of particular math skills that would be useful?

6. Math practice outside of this class time will be strongly suggested for two reasons. One is that it is often difficult to remember the topic covered in a class from week to week and is often very difficult to remember the skills taught without some practice between classes. The other reason is that some people may want or need math practice on skills not covered in class or may need or want extra practice in some areas. Please circle below your response to out-of-class practice (commonly known as homework).

I'd like to do lots!

I'm happy to do anything the instructor thinks would be helpful.

I'm willing to do what's necessary.

I'll do some if I have to.

No way!

MATH PRACTICE LOG

Date	11/20 - 11/27	11/27 - 12/4	12/4 - 12/11	12/11 - 12/18	12/18 - 1/8	1/8 - 1/15	1/15 - 1/22	1/22 - 1/29	1/29 - 2/5	2/5 - 2/12	2/12 - 2/19
Practiced calculation with whole numbers (+, -, ×, ÷)											
Practiced calculation with decimals (+, -, ×, ÷)											
Explained math from this class to a family member											
Explained math problem to my child or other student											
Did my math homework											
Problem-solved at work using math											
Problem-solved at home using math											
Practiced math skills on computer/LearningPlus											
Practiced an unfamiliar calculator operation											
Wrote up & brought to class a typical work use of math											
Brought to class a work use of math I didn't understand											

**Please make notes on the back of this sheet about specific problems you solved or other information that won't fit in the boxes.

Ground Rules

1. Everyone is smarter than he/she thinks he/she is, and everyone has something to contribute.
2. You cannot fail this class.
3. All people and all ideas are respected.
4. You know considerably more about your company and your work than your teacher does.
5. Teamwork is the foundation of learning.
6. Eating and drinking in class are fine. We all need a little extra energy.
7. Bring your calculator, paper, pencil, and good sense to each class.
8. Listening is as important as speaking.
9. Participation is expected, but you can always "pass" or say "I don't know."
10. No grades will be given in this class.
11. Bring any day-to-day math problems or errors encountered on the job for discussion in class.
12. There are fewer wrong answers than you think.
13. Respectfully challenging ideas is encouraged. (Including the ideas of the instructor.)
14. You can sketch at least as well as your instructor.
15. Confidentiality is a must. It is important not to share information about other students.
16. You instructor may spend a lot of time looking over your shoulder. This is not meant to make you nervous. it is meant to offer help and new ideas early-- before you get entrenched in a mistake or frustrated and confused by the way you are doing something. (Also, it is a way that I pick up lots of new idea.)
17. Enjoy yourself! If you're not having fun you are probably not learning.

MATH ANXIETY BILL OF RIGHTS

(SOURCE UNKNOWN)

I HAVE A RIGHT TO LEARN AT MY OWN PACE AND NOT
FEEL PUT DOWN OR STUPID.

I HAVE A RIGHT TO ASK WHATEVER QUESTIONS I HAVE.

I HAVE THE RIGHT TO NEED EXTRA HELP.

I HAVE THE RIGHT TO SAY I DO NOT UNDERSTAND.

I HAVE THE RIGHT TO FEEL GOOD ABOUT MYSELF
REGARDLESS OF MY ABILITIES IN MATH.

I HAVE THE RIGHT NOT TO BASE MY SELF-WORTH ON
MY MATH SKILLS.

I HAVE THE RIGHT TO VIEW MYSELF AS CAPABLE OF
LEARNING MATH.

I HAVE THE RIGHT TO EVALUATE MY MATH INSTRUCTORS
AND HOW THEY TEACH.

I HAVE THE RIGHT TO RELAX.

I HAVE THE RIGHT TO BE TREATED AS A COMPETENT
ADULT.

I HAVE THE RIGHT TO DISLIKE MATH.

I HAVE THE RIGHT TO DEFINE SUCCESS IN MY OWN
TERMS.

OPEN CALCULATION

For each problem below fill in the empty squares with numbers so that the final set-up is a correct subtraction, multiplication or division problem.

$$\begin{array}{r} \square 94\square \\ - 28\square 3 \\ \hline 8\square 26 \end{array}$$

$$\begin{array}{r} \square\square\square \\ \times 26 \\ \hline 2304 \\ \square\square\square \\ \hline \square\square\square 4 \end{array}$$

$$\begin{array}{r} 4\square\square \\ \square\square \\ \hline 19\square\square \\ 3\square 0\square \\ \hline \square\square 9\square\square \end{array}$$

$$\begin{array}{r} \square\square \overline{) 2\square\square 2} \\ \underline{\square 6} \\ 5\square \\ \underline{\square\square} \\ 0 \end{array}$$

$$\begin{array}{r} \square\square 2\square 4 \\ - 5\square 2\square \\ \hline 4539 \end{array}$$

In each problem below, each different letter stands for a different number. Find numbers to replace the letters so that each set-up becomes a correct addition.

$$\begin{array}{r} MA \\ + A \\ \hline AM \end{array}$$

$$\begin{array}{r} Y \\ + Y \\ + Y \\ \hline MY \end{array}$$

Number Patterns, Problem-solving and Review

1. Consider the numbers from 1 to 100. Suppose you circled all the multiples of 4, put a box around all the multiples of 6, and underlined all the multiples of 9. Which numbers would be circled, boxed, and underlined?

2. The sum of any two-digit number and the number formed by reversing the digits is always divisible by _____.

3. If it is 7 o'clock a.m., what time will it be in 11, 999,999,994 hours?

4. Assuming D stands for the same number each time it is used in this equation:
 $D \times D = D + D$, what number could replace D and make the equation true?

Find the missing numbers in each of the following sequences.

5. 7, _____, 21, 28, 35, _____, _____

6. 2, 16, 4, 32, 8, _____, 16, 128, _____

7. 1, 4, 9, 16, _____, 36, 49, _____

Finding Averages
(also called the mean or arithmetic mean)

Finding the average (or mean) of a group of numbers is a way of showing a general picture of specific information. An average shows information spread out evenly rather than in the clusters it may actually have come in. To compute an average of a group of numbers, total the numbers, then divide by the number of numbers you added.

For instance, if a part-time worker worked 8 hours on Monday, 3 hours on Tuesday and none on Wednesday, his average time worked per day would be figured by adding $8 + 3 + 0$ (giving 11) and dividing by 3.

Try the following problems involving averages. You can use a calculator for the computations(unless otherwise indicated), but please show on your paper the steps you went through to get your answer.

1. During one week, a pipe company produced the following amounts of pipe:

Monday: 150,000 pounds	Tuesday: 148,600 pounds
Wednesday: 139,450 pounds	Thursday: 151,100 pounds
Friday: 142,750 pounds	Saturday: 145,000 pounds
Sunday: 147,200 pounds	

What was the average weight of pipe produced per day during that week?

2. During that same week, the following amount of pipe was rejected:

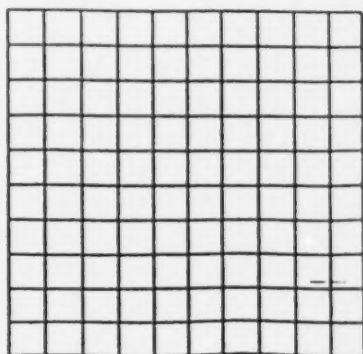
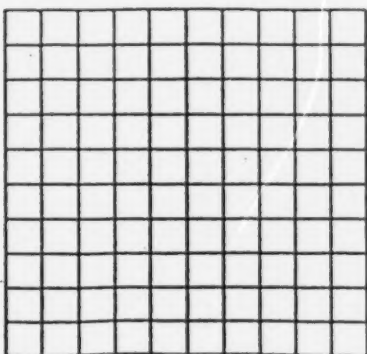
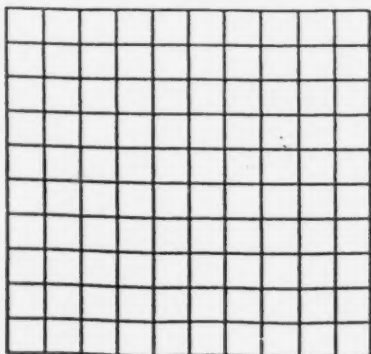
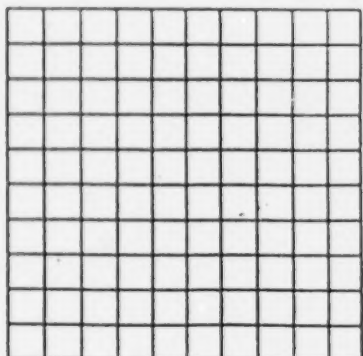
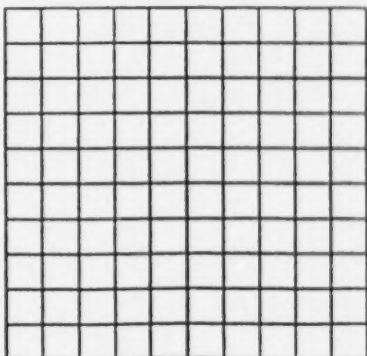
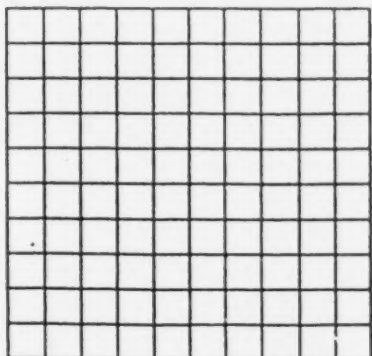
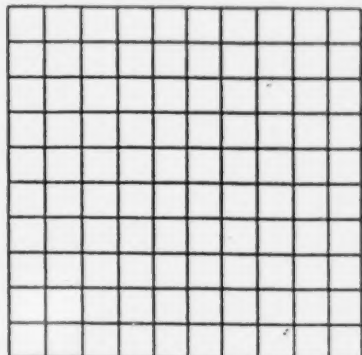
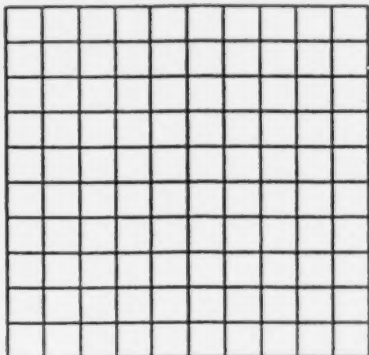
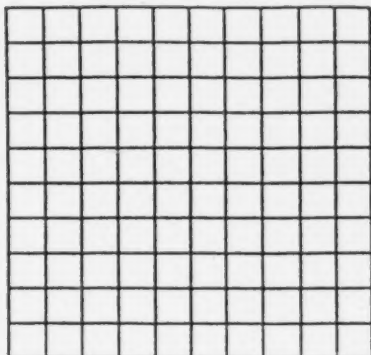
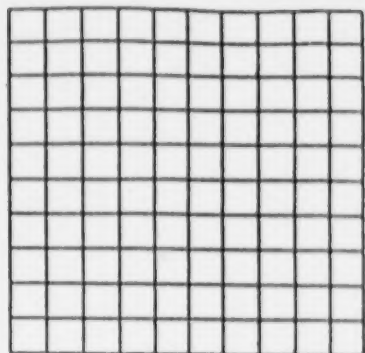
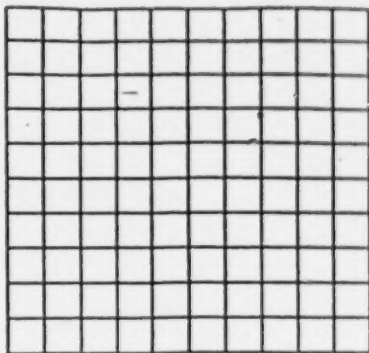
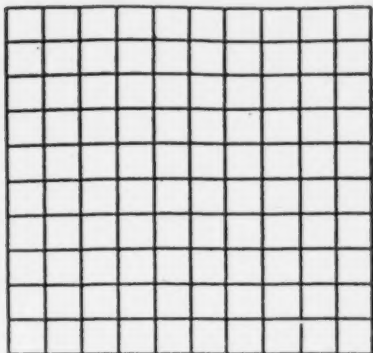
Monday: 110 pounds	Tuesday: 705 pounds	Wednesday: 75 pounds
Thursday: 1100 pounds	Friday: 630 pounds	Saturday: 0 pounds
Sunday: 350 pounds		

What was the average weight of pipe rejected per day during that week?

3. The average annual rainfall in a certain area (not the Willamette Valley) is 7 inches per year. About how much rainfall is this per month? (Note the word "about". I do not want an exact answer. I want a quick and easy estimate that you do in your head--not on the calculator.)

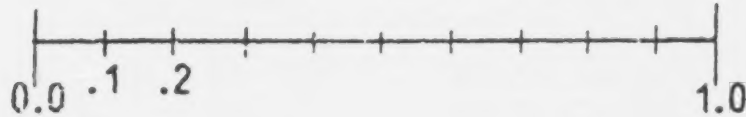
4. Write a pair of numbers whose average is 25.

5. Write 2 different sets of 3 numbers whose average is 17.



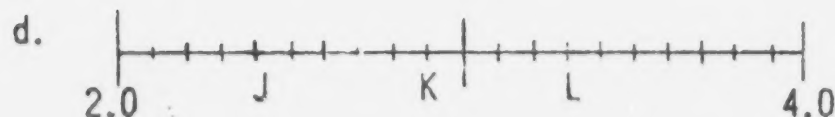
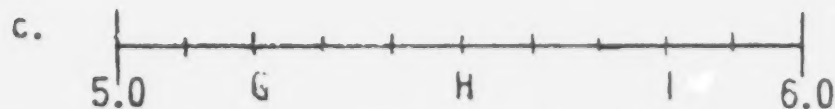
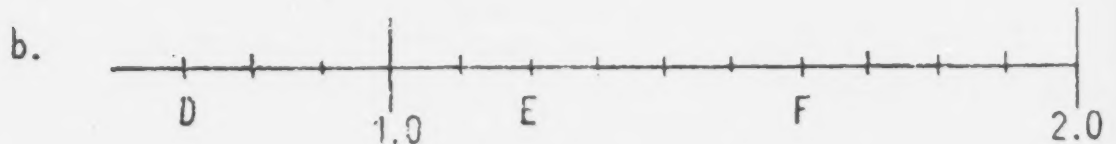
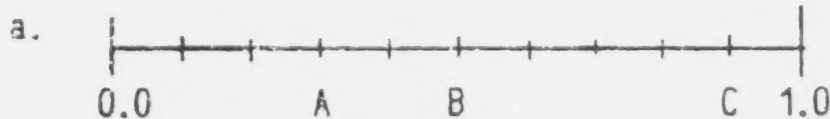
NUMBER LINE DECIMALS

Number lines can help us understand decimals. This segment is divided into 10 equal parts.



1. Some marks are labeled. Label the other marks.
2. Write the decimal name for the 0 mark. _____
3. Write the decimal name for the 1 mark. _____
4. Parts of these number lines are lettered. Label each letter with its decimal name.

Look closely to see how each number line is different.



PRACTICE

DECIMALS

Skill 1

page 1

Read each decimal and write it in the place-value chart.
Remember AND separates the whole number from the decimal.

ANSWERS

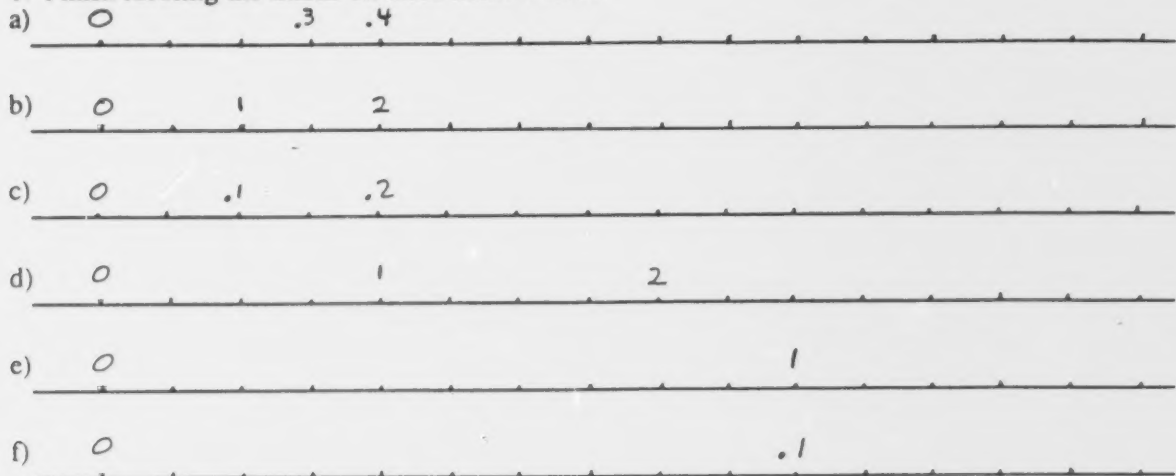
1. 9.4
2. 9.53
3. 14.126
4. 68.4
5. 206.427
6. 104.9
7. 346.76
8. 147.204
9. 9.03
10. 0.8
11. 0.27
12. 0.07
13. 0.0117
14. 68.04
15. 0.017
16. 0.007
17. 206.0027
18. 206.007
19. 14.026
20. 14.0006
21. 60.004
22. 555.047
23. 263.0003

		hundreds	tens	ones	.	tenths	hundredths	thousandths	ten-thousandths
1.	9 and 4 tenths								
2.	9 and 53 hundredths								
3.	14 and 126 thousandths								
4.	68 and 4 tenths								
5.	206 and 427 thousandths								
6.	104 and 9 tenths								
7.	346 and 76 hundredths								
8.	147 and 204 thousandths								
9.	9 and 3 hundredths								
10.	8 tenths								
11.	27 hundredths								
12.	7 hundredths								
13.	117 ten-thousandths								
14.	68 and 4 hundredths								
15.	17 thousandths								
16.	7 thousandths								
17.	206 and 27 ten-thousandths								
18.	206 and 7 thousandths								
19.	14 and 26 thousandths								
20.	14 and 6 ten-thousandths								
21.	60 and 4 thousandths								
22.	555 and 47 thousandths								
23.	263 and 3 ten-thousandths								

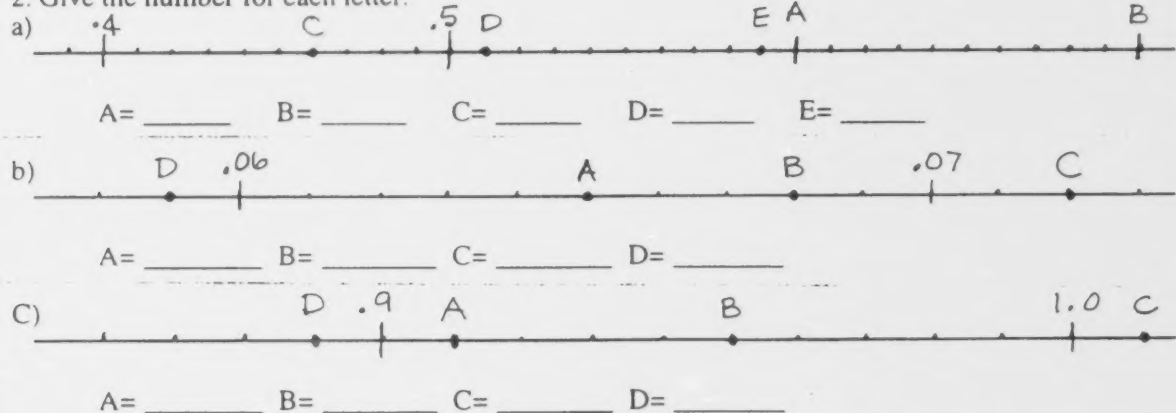
Decimal Sense Name _____

Decimals on a number line

1. Finish labeling the marks on these number lines.



2. Give the number for each letter.



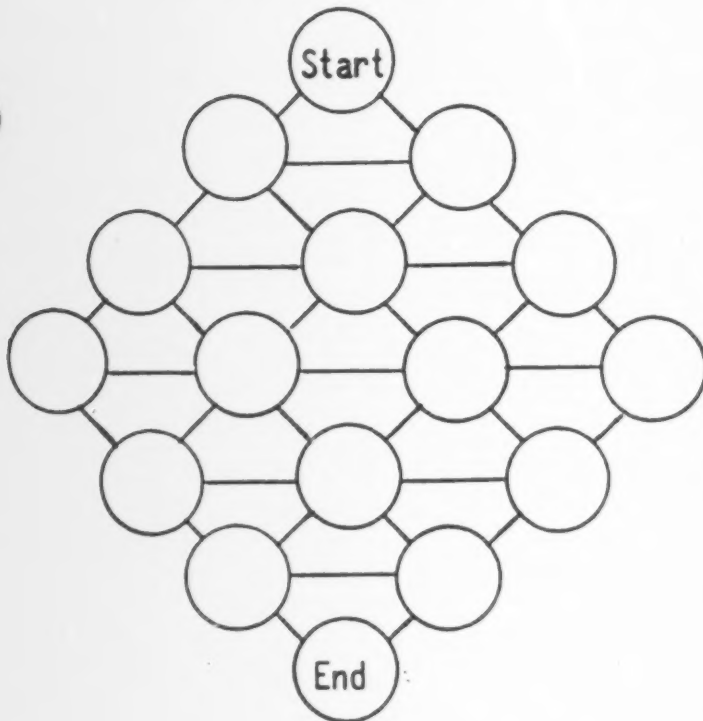
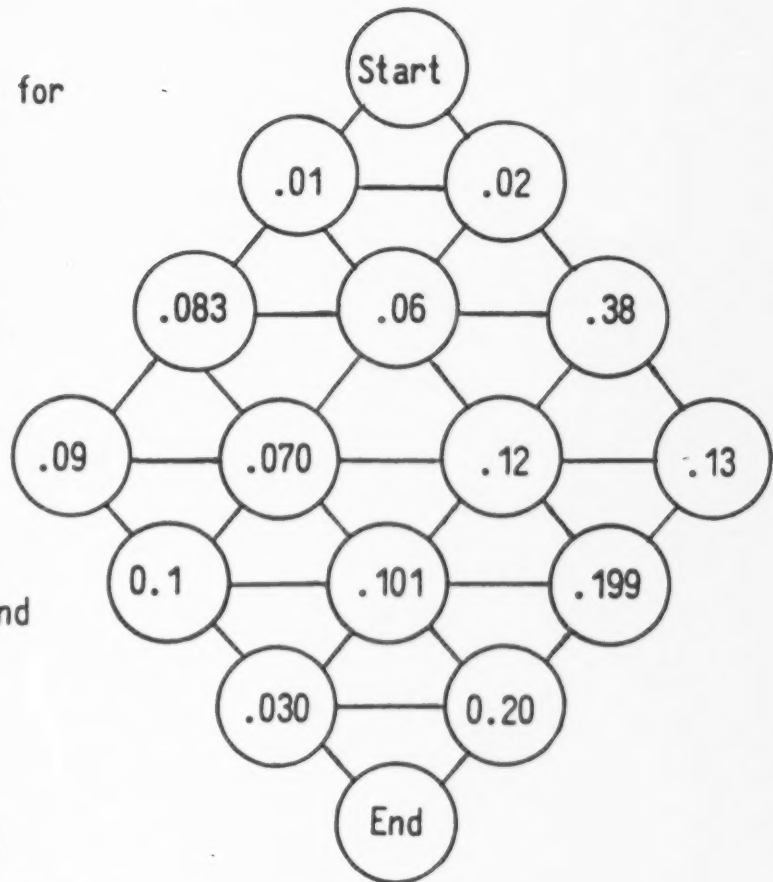
3. Multiplying by 10, 100, 1000

$14.6 \times 10 =$ _____	$16.7 \times 100 =$ _____	$.3 \times 1000 =$ _____
$.7 \times 10 =$ _____	$6.0 \times 100 =$ _____	$.017 \times 1000 =$ _____
$3.00 \times 10 =$ _____	$83,204 \times 100 =$ _____	$219.6 \times 1000 =$ _____
$3.4 \times$ _____ $= 34$	$.04 \times$ _____ $= 4$	$5 \times$ _____ $= 5000$
$.8 \times$ _____ $= 800$	$.07 \times$ _____ $= .7$	$.7 \times$ _____ $= 70$
_____ $\times 10 = 17.8$	_____ $\times 100 = .8$	_____ $\times 1000 = 400$

GOING IN CIRCLES

Rules

- . Don't count the circles for "start" and "end."
 - . Always move to a larger number.
1. Find the longest path through the maze.
 2. There are four different paths that are the shortest. Find all four of them.



3. Find a path that is exactly six circles long.
4. Find a path that is exactly nine circles long.
5. Which circles will never be used? Explain.
6. Design your own maze. Give it to a classmate to work.

Rules for Computing with Decimals(1)

Below you will find some typical decimal additions. Use these as examples to determine the rules for adding decimal numbers. Then write those rules on the "Rules" lines. Finally, on the "Things to Watch For" lines, note any "tricky" parts of decimal addition--that is, what's the easiest thing to forget or skip over, what's most confusing, etc.

ADDITION:

$$.58 + .9 =$$

$$2.0075 + .004 + 35.8 + .00128 =$$

$$9.2 + .108 + 441 + 20.11 =$$

RULES:

WHAT TO WATCH FOR:

Rules for Computing with Decimals(2)

Below you will find some typical decimal subtractionss. Use these as examples to determine the rules for subtracting decimal numbers. Then write those rules on the "Rules" lines. Finally, on the "Things to Watch For" lines, note any "tricky" parts of decimal subtraction--that is, what's the easiest thing to forget or skip over, what's most confusing, etc.

SUBTRACTION:

$36.29 - 12.03 =$

$.6 - .25 =$

$478.33 - 2.5 =$

$15 - .0028 =$

RULES:

WHAT TO WATCH FOR:

Placing the Point 1

Use estimation to put the decimal points in the correct places. Add extra zeros if needed. Check with your calculator after each group.

Example: $1.8 \times 1.5 = 2.700$

Think — 1.8 is almost 2. Since 1.5 times 2 is 3, the answer must be a little less than 3. That means the decimal point should be between the 2 and the 7.

1. 12. ☐ 15. ☐
0.12 ☐ 0.15 ☐
0.12 ☐ 1.5 ☐
1.2 ☐ 1.5 ☐

2. 26. ☐ 25. ☐
2.6 ☐ 2.5 ☐
0.026 ☐ 2.5 ☐
0.26 ☐ 0.25 ☐

3. 8. ☐ 35. ☐
0.8 ☐ 0.35 ☐
0.08 ☐ 0.035 ☐
0.8 ☐ 3.5 ☐

4. 14. ☐ 55. ☐
1.4 ☐ 5.5 ☐
0.14 ☐ 0.55 ☐
1.4 ☐ 0.55 ☐

5. 12. ☐ 45. ☐
0.12 ☐ 4.5 ☐
1.2 ☐ 4.5 ☐
0.12 ☐ 0.45 ☐

6. 7. ☐ 15. ☐
0.7 ☐ 1.5 ☐
7. ☐ 1.5 ☐
0.7 ☐ 0.15 ☐

Rules for Computing with Decimals(3)

Below you will find some typical decimal multiplications. Use these as examples to determine the rules for multiplying decimal numbers. Then write those rules on the "Rules" lines. Finally, on the "Things to Watch For" lines, note any "tricky" parts of decimal multiplication--that is, what's the easiest thing to forget or skip over, what's most confusing, etc.

MULTIPLICATION:

$26 \times .11 =$

$3.8 \times 32.61 =$

$.0212 \times 1.003 =$

$.0023 \times .014 =$

RULES:

WHAT TO WATCH FOR:

Hit the Target

Arrange the digits 1, 2, 3, 4, 5, and 6 in the boxes to make two decimal factors whose product comes as close to the target as possible.

Subtract your product from the target and write the difference as your score. The total is your game score.

1. Target 30

$$\begin{array}{r} \square . \square \square \\ \times \square \square . \square \\ \hline \end{array}$$

Your product _____
Your score _____

2. Target 1000

$$\begin{array}{r} \square . \square \square \\ \times \square \square . \square \\ \hline \end{array}$$

Your product _____
Your score _____

3. Target 15

$$\begin{array}{r} \square . \square \square \\ \times \square . \square \square \\ \hline \end{array}$$

Your product _____
Your score _____

4. Target 250

$$\begin{array}{r} \square . \square \square \\ \times \square . \square \square \\ \hline \end{array}$$

Your product _____
Your score _____

5. Target 35

$$\begin{array}{r} \square . \square \square \\ \times \square \square . \square \\ \hline \end{array}$$

Your product _____
Your score _____

6. Target 20

$$\begin{array}{r} \square . \square \square \\ \times 0 . \square \square \square \\ \hline \end{array}$$

Your product _____
Your score _____

7. Target 300

$$\begin{array}{r} \square \square . \square \\ \times 0 . \square \square \square \\ \hline \end{array}$$

Your product _____
Your score _____

8. Target 300

$$\begin{array}{r} \square \square . \square \\ \times \square . \square \square \\ \hline \end{array}$$

Your product _____
Your score _____

Game Score _____

Rules for Computing with Decimals(4)

Below you will find some typical decimal additions. Use these as examples to determine the rules for subtracting decimal numbers. Then write those rules on the "Rules" lines. Finally, on the "Things to Watch For" lines, note any "tricky" parts of decimal subtraction--that is, what's the easiest thing to forget or skip over, what's most confusing, etc.

DIVISION:

$$52.008 \div 8 = \quad .0168 \div 7 = \quad 8.4 \div .2 = \quad .338 \div .13 = \quad 6 \div .4 =$$

RULES:

WHAT TO WATCH FOR:

Decimal Estimation 2

Try to think of a whole number that will give an answer within the range. Enter your guess, press $\div$, and enter the factor. You have three tries to make it into the range.

1. _____ $\div$ 1.3 $\div$
Second try _____ $\div$
Third try _____ $\div$
Range 50 55

2. _____ $\div$ 4.3 $\div$
Second try _____ $\div$
Third try _____ $\div$
Range 50 55

3. _____ $\div$ 0.7 $\div$
Second try _____ $\div$
Third try _____ $\div$
Range 50 55

4. _____ $\div$ 0.3 $\div$
Second try _____ $\div$
Third try _____ $\div$
Range 50 55

5. _____ $\div$ 0.61 $\div$
Second try _____ $\div$
Third try _____ $\div$
Range 50 55

6. _____ $\div$ 0.84 $\div$
Second try _____ $\div$
Third try _____ $\div$
Range 50 55

7. _____ $\div$ 0.08 $\div$
Second try _____ $\div$
Third try _____ $\div$
Range 50 55

8. _____ $\div$ 0.048 $\div$
Second try _____ $\div$
Third try _____ $\div$
Range 50 55

WHICH ANSWER IS REASONABLE?

- 1) $4567 \times 89 =$
(4,265,896 * 416,492 * 39,103 * 41,073 * 405,863 * 406,463 * 415,963 * 396,393)
- 2) $6982 \div 97$ (divide to 2 decimal places) =
(7.19 * 794.86 * 62.55 * 910 * 71.98 * 7.65 * 72.88 * .710)
- 3) Round 3.278 to the nearest hundredth (2 decimal places) =
(3.28 * 3.280 * 3.3 * 3.300 * 14.072)
- 4) $42.006 + 9 + .1125 =$
(42.015 * 42.1275 * 51.1185 * 61.1185)
- 5) $342 + 2.4 + .0003 + 95.011 =$
(131.6113 * 439.4113)
- 6) $19 - 4.028 =$
(4.009 * 14.972 * 15.028 * 4.972 * 14.72)
- 7) $9 - 2.6015 =$
(6.3985 * 7.6015)
- 8) $3.027 \times .05 =$
(156.35 * 1,513 * .16035 * 15.035 * .15135 * 1513.5)
- 9) $31.9 \times .002 =$
(638 * .638 * .06 * .0638)
- 10) $.0672 \div 1.2 =$
(.056 * 56 * .560)
- 11) $4.662 \div 50 =$
(932 * .932 * .09324 * 93.204 * .009324 * 9.324)
- 12) $52.9 \div .002 =$
(2641 * .2645 * 26,850 * 2.645 * 26,450 * 6.45 * 26.45)
- 13) Convert .875 to a fraction.
($15/16$ * $5/8$ * $7/8$ * $1-3/4$)
- 14) Convert $3/8$ to a decimal.
(.45 * .375 * .38)
- 15) Convert $7/8$ to a decimal.
(.875 * 1.1 * .825)

BE REASONABLE

1. Circle the most reasonable answer for each problem. No correct answers are given so do not try to work the problems out.

- a. $38 \times .8$
- b. $12.2 \div 4.1$
- c. 2.9×5.2
- d. $19.76 \div .43$
- e. 83.9×1.03
- f. $57 \div .89$
- g. $.76 \times 9.2$
- h. $4.6 \div 9.2$
- i. $9.27 \times .52$
- j. $4.23 \div 2.4$

a	b	c
300	40	3.8
50	30	3
16	1500	8.1
0	15	40
9	85	839
5.8	58	580
8	63	10
20	.4	36
50	5	.5
200	20	2.0

2. Fill in each blank with a decimal that will give the reasonable answer shown. $\approx$ means "is about equal to."

- a. _____ $\times 5.2 \approx 30$
- b. _____ $\div 1.1 \approx 8.9$
- c. $36.9 \times$ _____ ≈ 20
- d. $78 \div$ _____ ≈ 80
- e. _____ $\times .96 \approx 50$
- f. $100 \div$ _____ ≈ 26

3. Each reasonable answer below has two choices for the problem. Circle the problem that best gives the reasonable answer.

	<u>a</u>	<u>b</u>		<u>a</u>	<u>b</u>				
a.	50	$50 \times .89$	40	$40 \times .89$	d.	36	$39 \div .86$	39	$39 \div 1.04$
b.	60	$100 \div 1.2$	80	$80 \div 1.2$	e.	25	5.1×5.4	5.1	5.1×4.9
c.	30	42.3×1.14	42.3	$42.3 \times .75$	f.	11	$90 \div 10.2$	90	$90 \div 9.8$

Decimal Review

Here is rather mixed up set of instructions for doing calculations with decimals. Match them to the decimal calculations on the next page.

- a. Add zeros at the end to show the same number of decimal places; then compare.
- b. Be sure you put the correct number on top. Line up the decimal points in the problem. Calculate; then put the decimal point in your answer directly under those decimal points.
- c. Set up the problem and do the calculations as if you were working with whole numbers. Place the decimal point in your answer so that there are exactly as many decimal places in your answer as in the total number of decimal places in the numbers in the problem.
- d. Read the decimal with its label (tenths, hundredths, thousandths, etc.). Write a fraction that would be read the same way. Reduce.
- e. Line up the decimal points in the problem and place the decimal point in your answer directly in line with those decimal points.
- f. Be sure you get the numbers in the right places. (Remember that $17 \div 21$ is the same as $21 \overline{)17}$.) If you are dividing by a whole number, just place the decimal in your answer directly above the decimal point under the division sign; then do the calculations. (Remember, too, that you must show an answer for each position after the decimal point.)
- g. We usually don't write the decimal point in a whole number, but remember that it's lurking invisibly in the spot that would separate the whole number from any decimal parts.
- h. Be sure that the correct number is on top with the decimal points lined up. Add zeros at the end to help keep things clear. Borrow as you normally would, if necessary. Decimal point in your answer should line up with those in the problem.
- i. Be sure you get the numbers in the right places. (Remember that $17 \div 21$ is the same as $21 \overline{)17}$.) If you are not dividing by a whole number, move the decimal point in that number (the divisor) to the right until it becomes a whole number. Then, in the number to be divided, move the decimal point the same number of places to the right. Now, place the decimal point for your answer directly above this new decimal point position and do the calculations. (You must show an answer for each position after this decimal point.)
- j. Divide the top number by the bottom number. (The top number goes inside the division sign.) Add a decimal point and one or more zeros to the number you're dividing. Divide. Stop when the answer come out with no remainder. If this doesn't happen, stop when you've divided 4 places after the decimal; then, round off the answer to thousandths (3 decimal places).

NAME _____ DATE _____

Secret Message Using Decimals

The answer to each problem in the list below, when rounded off to the nearest whole number, represents a letter of the alphabet. When all the letters have been assembled in respective order, they form the words of a message. Using the alphabet-number chart to decode the letters, see if you can determine the secret message.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

- _____ Which is the smallest: 21.3, 31.2, 132, or 13.2?
- _____ Round off .09 to the nearest number of 12ths.
- _____ $4.25 + .037 + 13.7 + .576 + 1.208$
- _____ $14.28 - 5.9$
- _____ 1.78×5.23
- _____ $26.22 \div 1.38$
- _____ How many times is .3125 contained in $\frac{5}{16}$?
- _____ Find the sum of .6, 3.06, .826, 1.057, and 14.3.
- _____ Subtract 5.837 from 21.05.
- _____ Determine the product of 2.18 and 6.945.
- _____ If 47.6 is the dividend and 3.87 is the divisor, find the quotient.
- _____ 5.4 is .36 of what number?
- _____ $.15 \times 9.6 \div .24$
- _____ Multiply $14\frac{3}{4}$ by 1.59.
- _____ Subtract $6\frac{3}{8}$ from the product of 2.4 and 6.5.
- _____ Divide the sum of 8.02 and $132\frac{1}{2}$ by 7.38.
- _____ .76 of what number is 3.04?
- _____ $2.1 \times 1\frac{1}{4} \div .175$
- _____ $\frac{2.6 \times 1.4 \times .15}{.012 \times 3.5}$



Deciding When to Add, to Subtract, to Multiply, or to Divide

To solve real mathematical problems, the first step is to decide which operation to use. This process often seems more difficult when working with fractions, although the thinking involved is very similar whether the problem involves fractions, decimals, whole numbers or a combination of these.

You **add** when you need to **combine amounts together** or when you need a **total** or a **sum**.

Example: Jane earned \$1,428 in January, \$1,902 in February, and \$1,566 in March. How much had she earned by the end of March?

Example: Rainfall for the first four months of the year was $2\frac{1}{2}$ ", $3\frac{1}{4}$ ", $1\frac{1}{8}$ ", and $1\frac{1}{2}$ ". What was the total rainfall for those months?

You **subtract** when you need to know the **amount of difference between two amounts** or when **one amount is being taken from another amount**.

Example: City A has a population of 47,231 while City B's population is 86,495. How much greater is the population of City B than City A?

Example: The storekeeper sold $9\frac{1}{4}$ yards from a piece of material containing 53 yards. How much was left in the original piece?

You **multiply** when you need to **combine equal groups** or when you need to **find a fractional part of** something.

Example: Sue rides her bike 4.5 miles every day. How many miles does she ride in a 30-day month?

Example: Beth works $5\frac{1}{4}$ hours per day. How many hours does she work in a 5 day week?

Example: Bert's gas tank holds 14 gallons. If he has $\frac{3}{4}$ tank of gas, how many gallon is that?

You **divide** when you need to **divide something into a specified number of groups** or when you need to **separate something into groups of equal size**.

Example: Tobi has \$3285 that she wants to divide equally among her 3 children. How much should each child receive?

Example: Austin was selling carrots from his garden in packages of 2.25 pounds each. How many packages would he have if he packaged 71 pounds of carrots today?

Beginning Word Problems with Decimals

For each of the following problems, decide what math operation you will need to use to solve the problem. Write down the necessary calculation. Compute.

1. Esther gave the clerk a twenty-dollar bill for a purchase of \$14.51. How much change should she have received?

2. A jet flew averaged 499.4 miles per hour in a flight from New York to London that took 7 hours. How many miles was the trip?

3. Chicago to Memphis is 527.4 miles. Memphis to New Orleans is 393.8 miles. How far is Chicago from New Orleans?

4. A shipping box contains 24 cans of degreaser. If each can weighs 2.5 pounds, what is the total weight of the contents of the box?

5. The price of a 3.5-pound package of chicken is \$4.69. What is the cost of the chicken per pound?

6. The storekeeper sold 9.25 yards from a piece of fabric containing 53 yards. How much was left in the original piece?

7. Rainfall last month was 2.4 inches, .19 inch, 3 inches, 1.95 inches. What was the total rainfall for the month?

8. Hank stacked 3 washers to use as a spacer. What is the width of Hank's spacer if each washer is 0.0625 inch wide?

Word Problems with Decimals

Use your decimal computation skills and your problem-solving skills to answer the following questions. Show your work. Circle your answer.

1. The length of the United States flag is 2.9 times its width. How long should a flag be if its width is 5 feet?
2. In the deli, cheese sells for \$5.29 per pound. How much does .75 pound cost?
3. What is the maximum length that is acceptable if a tolerance of .005 inch is permitted on pipe designed to be 1.625 thick?
4. To find the batting average, divide the number of hits by the number of times at bat, finding the quotient(answer) to the nearest thousandth. What is Jan's batting average if she had 11 hits out of 35 times at bat?
5. Normal atmospheric pressure is 29.92 inches of mercury. How much above normal is a barometric pressure of 30.39?
6. Workers in manufacturing industries averaged 40.3 hours of work and earnings of \$49.25 per week in 1947. In 1961 the work week averaged 39.8 hours and earnings increased to \$92.34. How many times as much were workers earning per week in 1961 as in 1947?
7. At the same rate of increase as in problem #6, what would you expect workers to be earning per week in 1975?
8. Again, at the same rate of increase, what would you expect manufacturing workers to be earning per week in 1999?
9. All but 17.7 pounds of topsoil were sold at a sale. if the store began with 100 pounds of the topsoil and charged \$.59 a pound, how much money did it take in?

How many calories do I burn if I . . . ?

Use the following formula to find out how many calories you burn during different activities:

Calories/minute x your weight x minutes you exercise.

Activity	Calories/Minute/Lb
Chopping wood	.135
Squash	.096
Running (8 min.mile)	.096
Racquetball	.081
Snowshoeing (soft snow)	.075
Rope jumping (70 jumps/min.)	.074
Skiing (cross-country, fast)	.064
Stair climbing	.056
Rowing machine	.046
Aerobic dancing (med. pace)	.046
Cycling (9.5 mph)	.045
Weight lifting (circuit trng.)	.041
Stacking wood	.040
Weight lifting (free weights)	.039
Walking (normal pace)	.036
Food shopping	.028
Sweeping, vacuuming	.028
Window cleaning	.026
Ballroom dancing	.023
Cooking	.022
Card playing	.011
Lying still	.010

DECIMAL PRACTICE WITH CALORIES

Use the information in the chart at left to answer the following questions. You may either use a standard weight of 180 lb. or indicate the weight you are using on this line _____

1. How many calories per pound does 1 minute of chopping wood burn?

2. How many calories would you burn in playing half an hour of racquetball?

3. How many calories would you burn in 1 hour of moderate cross-country skiing?

4. How many more calories would you burn in 30 minutes of running than you would in 30 minutes of normal walking?

5. How many more calories would you burn in 30 minutes of vigorous chopping wood than in 30 minutes of running?

6. How many calories would you burn if you spent a full 24 hours just lying still?

Decimal-Fraction Conversions

Name _____

Write a decimal or fraction equivalent for each number.

Fraction	Decimal	
$\frac{1}{8}$		
$\frac{3}{4}$		
	0.01	
$\frac{1}{6}$		
	0.625	
	0.85	
	1.45	
$\frac{2}{3}$		
	0.012	
$\frac{2}{9}$		
$\frac{1}{10}$		
$1\frac{1}{4}$		
	1.06	
	2.00	
	5.125	

Equal Fractions and Decimals

Circle the two numbers in each of the rows below that are equal. The first item has been completed for you as an example. Time is limited to 15 minutes.

1.	.001	①	①/10	1/100
2.	1/20	5.0	1/2	.05
3.	.25	1/2	1/4	.52
4.	2 1/2	.4	.25	2/5
5.	.6	3/4	.65	3/5
6.	7 1/2	.75	3/4	.075
7.	3.2	8/25	3 1/5	.165
8.	.027	27/10	27/100	2.7
9.	.19	19/10	1.9	9 1/10
10.	7.5	3/4	.075	7 1/2
11.	6.25	.625	6 1/2	5/8
12.	7/8	8.75	.875	3/4
13.	5 1/5	5.8	.58	5/8
14.	.625	1/32	.0625	1/16
15.	4.3	.43	43/1000	43/10
16.	5/8	6 3/8	6.375	.6375
17.	.125	1/16	1/8	.8
18.	1/6	.08 1/3	.625	1/12
19.	.16 2/3	1/6	.66 2/3	1/3
20.	.875	5/6	.83 1/3	2/3
21.	.032	3 1/5	.32	8/25
22.	1/30	1/3	3 1/3	.33 1/3
23.	.66 2/3	1 2/3	6.6	2/3
24.	3/200	.0015	.015	3/20
25.	26 1/2	2 1/8	2.625	26.25



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Scores:

0-4, Very Sad
9-12, Almost Sad
18-22, Glad

5-8, Sad
13-17, Almost Glad
23-25, Very Glad



Have Fun with Math

SAMPLE PROBLEMS WHEN RATIO AND PROPORTION CAN BE USED

PROBLEMS STATED AS RATIOS

Teacher Tips: Remind students that when they see the word *ratio* the problem is asking for a comparison. Ratios can be written with a colon (:) or as a fraction. It is customary to state a ratio in simplest terms, in just the same manner as simplifying or reducing fractions. It is important that the numbers are written in the same order as the words in the problem. Students might label the parts to remember the order. Some ratios compare different units. When this occurs, one unit must be changed to the other so that the units will be the same to compare.

In Jack's math class, there are 15 women and 10 men.

What is the ratio of women to men? $\frac{\text{women}}{\text{men}} = \frac{15}{10} = \frac{3}{2}$

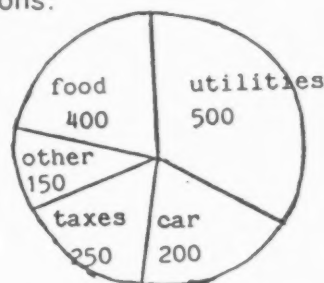
What is the ratio of women to total students in the class? $\frac{\text{women}}{\text{total}} = \frac{15}{25} = \frac{3}{5}$

What is the ratio of men to women? $\frac{\text{men}}{\text{women}} = \frac{10}{15} = \frac{2}{3}$

Look at the circle graph and answer the following questions:

What is the ratio of the amount spent on taxes to the amount spent on food?

$$\frac{\text{taxes}}{\text{food}} = \frac{250}{400} = \frac{5}{8}$$



What is the ratio of amount spent on utilities to the total budget?

$$\frac{\text{utilities}}{\text{total}} = \frac{500}{1500} = \frac{1}{3}$$

Find the ratio of 6 ounces to a pint. (NOTE: The units must be the same.)

1 pt. = 16 oz.

$$\frac{6 \text{ ounces}}{16 \text{ ounces}} = \frac{3}{8}$$

What is the ratio of 18 inches to a yard? (Remember: compare same units.)

36 in. = 1 yd.

$$\frac{18 \text{ in.}}{36 \text{ in.}} = \frac{1}{2}$$

PROBLEMS STATED AS RATIOS
WORKSHEET

Simplify these ratios:

1. 18 to 30
2. 4 to 16
3. 10 to 40
4. 21 to 14
5. 9 to 12
6. 50 to 25
7. Our kitchen is 18 feet long and 12 feet wide. What is the ratio of the width to the length?
8. On a test, Jerry got 25 problems right and 6 wrong. What is the ratio of the number he got right to the total number of problems?
9. Jane lost 50 pounds between March and June. She weighed 160 pounds in June. What is the ratio of the March weight to the June weight?
10. John bought a VCR for \$550. He saved \$100 by buying it on sale. What is the ratio of the sale price to the original price?
11. In the GED class, 25 are women and 20 are men. What is the ratio of men to women?
12. In the same GED class, what is the ratio of men to the total number of students?
13. The Stewarts make \$1050 a month. They pay \$450 for rent. What is the ratio of their rent to their income?
14. If the Stewarts pay \$500 for food, what is the ratio of their rent and food to their income?
15. A football team wins 7 out of 10 games played during the season. What is the ratio of the games lost to the number of games played?

**SAMPLE PROBLEMS
WHEN RATIO AND PROPORTION CAN BE USED**

GENERAL MULTIPLICATION/DIVISION WORD PROBLEMS

Teacher Tips: Show students how general multiplication and division word problems can be solved by setting up a proportion. Explain how to set up the proportion; remind students to be careful in comparing the items. Underline the key parts in the problem, and watch the units given. Again, it is a good idea to label the parts. It makes no difference how you compare things as long as you use the same method or order consistently. For example, if the top left number is in dollars, the top right number must also be in dollars. To set up the problems, use a ? or an X or a $\square$ for the missing part. To solve, find the cross product first and then remember that the cross products are equal, so to find the missing number you must divide.

If 4 oranges cost 60 cents, how much does 1 orange cost?

oranges	$\frac{4}{60}$	$\frac{1}{?}$	$1 \times 60 = 60$	$60 \div 4 = 15$
cents			$4 \times ? = 60$	

If a plane goes 720 miles in 3 hours, how far does it go in 1 hour?

miles	$\frac{720}{3}$	$\frac{?}{1}$	$720 \times 1 = 720$	$720 \div 3 = 240$
hours			$3 \times ? = 720$	

If 9 players spend \$63 for uniforms, how much does each player spend?

players	$\frac{9}{63}$	$\frac{1}{?}$	$63 \times 1 = 63$	$63 \div 9 = 7$
money			$9 \times ? = 63$	

If a car goes 162 miles on 6 gallons of gas, how far does it go on 2 gallons?

miles	$\frac{162}{6}$	$\frac{?}{2}$	$162 \times 2 = 324$	$324 \div 6 = 54$
gallons			$6 \times ? = 324$	

If a plane goes 900 miles in 3 hours, how far will it go in 8 hours?

miles	$\frac{900}{3}$	$\frac{?}{8}$	$900 \times 8 = 7200$	
hours			$3 \times ? = 7200$	

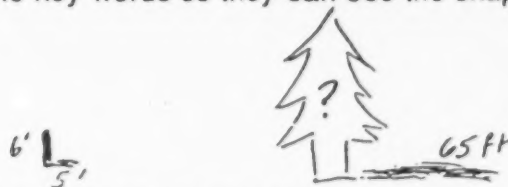
GENERAL MULTIPLICATION/DIVISION WORD PROBLEMS
WORKSHEET

1. If a car goes 90 miles in 2 hours, how far does it go in 1 hour?
2. If 2 cans of juice cost \$1.20, how much does 1 can cost?
3. If a 6-ounce can of crabmeat costs \$2.40, what is the cost per ounce?
4. If a car goes 336 miles in 6 hours, how many miles does it go per hour?
5. If 3 cans of spaghetti sauce cost \$2.67, how much will 5 cans cost?
6. If a man earns \$50 in 8 hours, how much does he earn in 5 hours?
7. If 4 ounces of cheese cost \$5.16, how much do 7 ounces cost?
8. If 1 out of every 6 seniors smoke, how many smoke out of 300?

SAMPLE PROBLEMS
WHEN RATIO AND PROPORTION CAN BE USED

SIMILAR SHAPES

Teacher Tips: Similar shapes share the same proportions. They are not necessarily the same size. It is a good idea to emphasize drawing the figures. Show how word problems take the shape of triangles and rectangles. Underline key words so they can see the shapes.



A six-foot tall vertical post casts a five foot shadow. At the same time, a tree casts a 65 foot shadow. How tall is the tree?

$$\begin{array}{l} \text{tall} \\ \text{shadow} \end{array} \quad \frac{6}{5} = \frac{?}{65} \quad \begin{array}{l} 65 \times 6 = 390 \\ 5 \times ? = 390 \end{array} \quad 390 \div 5 = 78$$

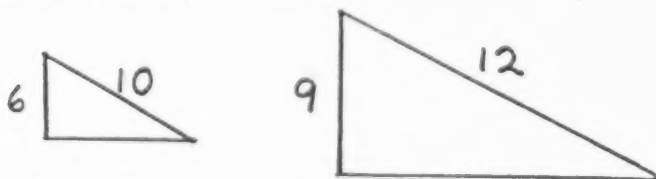
A snapshot is 5 inches wide and 7 inches long and is enlarged so that the width is 20 inches. How long is the enlargement?

$$\begin{array}{l} \text{wide} \\ \text{long} \end{array} \quad \frac{5}{7} = \frac{20}{?} \quad \begin{array}{l} 20 \times 7 = 140 \\ 5 \times ? = 140 \end{array} \quad 140 \div 5 = 28$$

A 6-foot vertical flagpole casts a shadow 4 feet long at the same time a building casts a shadow 72 feet long. How tall is the building?

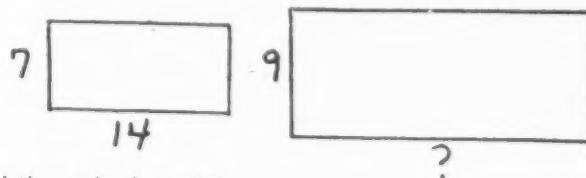
$$\begin{array}{l} \text{high} \\ \text{shadow} \end{array} \quad \frac{6}{4} = \frac{?}{72} \quad \begin{array}{l} 72 \times 6 = 432 \\ 4 \times ? = 432 \end{array} \quad 432 \div 4 = 108$$

Are these triangles similar?



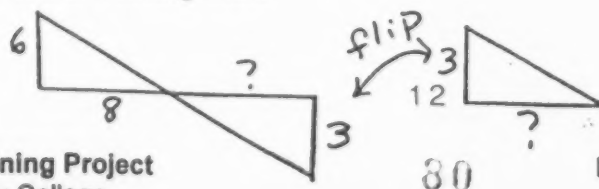
$$\frac{6}{10} = \frac{9}{12} \quad \begin{array}{l} 6 \times 12 = 72 \\ 10 \times 9 = 90 \end{array} \quad \begin{array}{l} 72 \neq 90 \\ \therefore \text{NO} \end{array}$$

Find the missing side in each rectangle.



$$\frac{7}{14} = \frac{9}{?} \quad \begin{array}{l} 14 \times 9 = 126 \\ 7 \times ? = 126 \end{array} \quad 126 \div 7 = 18$$

Find the missing side.



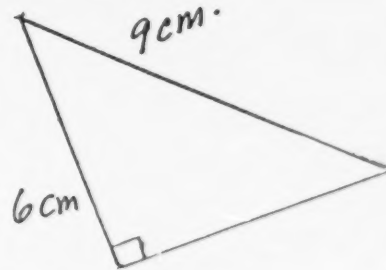
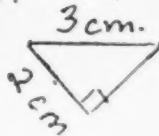
$$\frac{6}{8} = \frac{3}{?} \quad \begin{array}{l} 8 \times 3 = 24 \\ 6 \times ? = 24 \end{array} \quad 24 \div 6 = 4$$

SIMILAR SHAPES

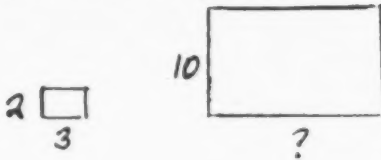
WORKSHEET

1. A six-foot vertical flagpole casts a shadow 4 feet long at the same time a building casts a shadow 72 feet long. How tall is the building?
2. A picture is 4 inches wide and 6 inches long and is enlarged so that the length is 9 inches. How wide is the enlargement?
3. A telephone pole casts a shadow 42 feet long at the same time a 5-foot tall man casts a shadow 10 feet long. How tall is the telephone pole?
4. The ratio of the width to the length of a snapshot is 4:5. If the snapshot is enlarged to 16 inches wide, how long will it be?

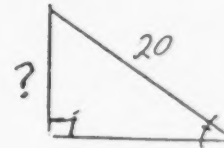
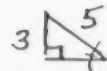
5. Are these triangles similar?



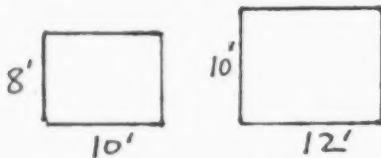
6. Find the missing side.



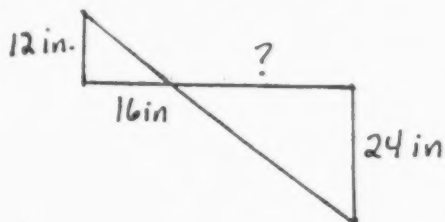
7. Find the missing side.



8. Are these figures similar?



9. Find the missing side.



Using Proportions: Estimation of Fish Population

At Green Lake, a fisheries research team tagged 120 fish for identification. Later they took a sample of 308 fish and found 8 tagged fish. Estimate the number of fish in the lake.

Write a proportion. Use a letter for the number you want to find.

$$\frac{120}{a} = \frac{8}{308}$$

Number of
tagged fish

Total number
of fish

Write the
cross-products.

$$120 \times 308 = a \times 8$$

$$36,960 = 8a$$

Find a .

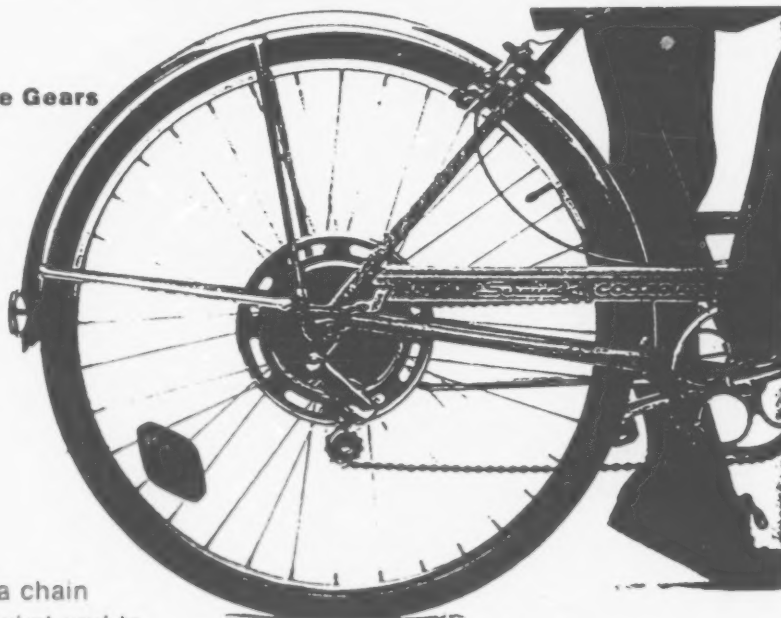
$$4620 = a$$

Answer the question.

The researchers estimated that there were about 4620 fish in Green Lake.

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Using Proportions: Bicycle Gears



Pat's 5-speed bicycle has a chain attached to the pedal sprocket and to the gears on the rear wheel. The ratio of pedal turns to rear-wheel turns for each gear is given below.

	1st gear	2nd gear	3rd gear	4th gear	5th gear
Number of pedal turns —————→	$\frac{9}{14}$	$\frac{4}{7}$	$\frac{1}{2}$	$\frac{3}{7}$	$\frac{5}{14}$
Number of rear-wheel turns —————→					

In first gear, how many times does the rear wheel turn if the pedals turn 180 times? (Use the ratio for first gear: 9 to 14.)

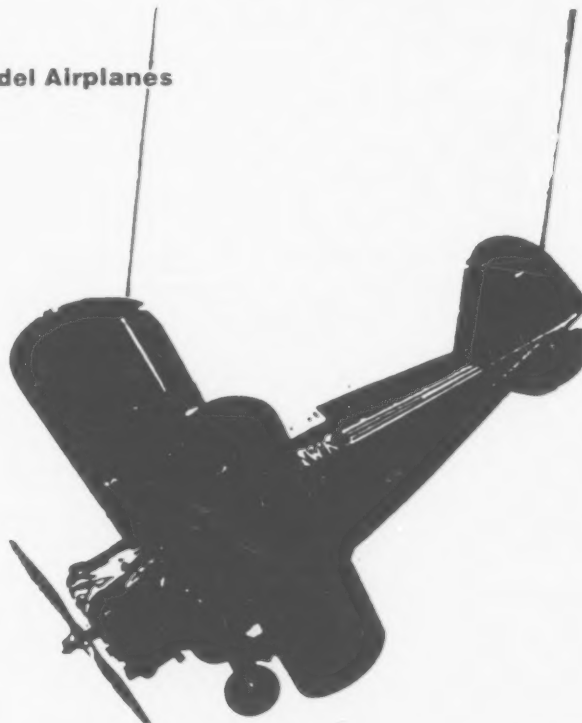
$$\frac{9}{14} = \frac{180}{n}$$

• Number of pedal turns
• Number of rear-wheel turns

$$n = 280$$

The rear wheel turns 280 times.

Using Proportions: Model Airplanes



- A. Teresa assembled a plastic model of the Grumman Gulfhawk stunt plane. The scale of this model is 1 to 32. One centimeter on the model represents 32 centimeters on the actual airplane.

The wingspan of the model is about 27 centimeters. What is the actual wingspan?

$$\frac{1}{32} = \frac{27}{n} \quad \leftarrow \text{Model airplane}$$
$$\quad \quad \quad \leftarrow \text{Actual airplane}$$

$$n = 864$$

The actual wingspan is about 864 centimeters, or 8.64 meters.

- B. The length of the Gulfhawk model is about 22 centimeters. What is the actual length of the airplane?

$$\frac{1}{32} = \frac{22}{n} \quad \leftarrow \text{Model airplane}$$
$$\quad \quad \quad \leftarrow \text{Actual airplane}$$

$$n = 704$$

The actual length is about 704 centimeters, or 7.04 meters

Practice with Ratios and Proportions

1. A baseball team won 14 games. If the ratio of games won to games played is 7 to 12, how many games has the team played?
2. A cleaning concentrate is to be mixed with water so that the ratio of water to concentrate is 8 to 1. At that rate, how much concentrate should be mixed with 16 quarts of water?
3. The ratio of boys of girls in the school is 11 to 10. If there are 330 boys at the school, how many girls are there at the school?
4. At the cafeteria, the ratio of pints of chocolate milk sold to pints of plain milk sold is 4 to 7. At this rate, how many pints of plain milk will be sold if 100 pints of chocolate milk are sold?
5. During the Ducks' last basketball game, the ratio of baskets made to baskets attempted was 2 to 5. If the Ducks made 26 baskets during the game, how many baskets did they attempt?
6. Last year the park department planted oaks and maples in a ratio of 5 to 7. If they planted 105 oaks last year, how many maples did they plant?
7. On a spelling test, the ratio of the number of words James spelled correctly to the number of words he misspelled was 4 to 1. If he spelled 20 words correctly, how many words did he misspell?
8. The ratio of the number of times Steve gets to bat to the number of times he hits the ball is 5 to 3. If he is at bat 35 times, how many hits should he get?
9. A typist can type 4 pages of manuscript in 45 minutes. At this rate, how long would it take her to type 20 pages?
10. Canned soup is 3 cans for 89 cents. How much would a dozen cans cost?
11. The ratio of the number of women to the number of men working at JPH is 2 to 3. If the factory has 750 workers, how many are women?
12. A feed mixture of 5 pounds of bone meal, 3 pounds of ground limestone, and 2 pounds of salt is a good source of calcium and phosphorus for livestock. How many pounds of bone meal would be required to make 360 pounds of the feed?
13. A pipe 8 feet long weighs 14 pounds. How long is a piece of the same type of pipe weighing 70 pounds?

Direct Variation

We have been using proportions to solve a variety of different problems. So far all of these have been examples of direct variation. Direct variation means that if one variable changes, a second variable changes in the same way. For instance, the cost of a purchase would vary directly with the number of items bought. A proportion to represent this relationship could be set up in several ways. Using n to represent the original number of items, c for the corresponding cost, N for a different number of items and C for that corresponding cost, these would be:

$$\begin{array}{cc} \underline{n} & \underline{c} \\ N & C \end{array} \qquad \begin{array}{cc} \underline{n} & \underline{N} \\ c & C \end{array} \qquad \begin{array}{cc} \underline{N} & \underline{C} \\ n & c \end{array} \qquad \begin{array}{cc} \underline{c} & \underline{C} \\ n & N \end{array}$$

Set up a proportion in 4 different ways to solve the following problem:
The ratio of an object's weight on Earth to its weight on Neptune is 5:7. How much would a man who weight 150 pounds here weigh on Neptune?

(Optional: Note that you could also relate these facts by saying that the weight on Earth is $5/7$ times the weight on Neptune. Or you could divide seven by five to show that the weight on Neptune is $1-2/7$ times the weight on Earth.)

1. The amount of sand used in mixing mortar is directly proportional to the amount of lime used. If 2 bags of lime are mixed with 5 bags of sand, how much sand is mixed with 15 bags of lime in a shipment of this mortar?
2. When an electric current is 32 A (amperes), the electromotive force is 288 V(volts). Find the force when the current is 45 A if the force varies directly as the current.
3. How long is a 50-pound roll of wire which weighs .3 pounds per foot?
4. When an electric current is 20 amperes, the electromotive force is 100 volts. How many amperes of current would it take to produce a force of 380 volts?
5. Sixteen tons of sulfur are needed to make 49 tons of sulfuric acid. How many tones of sulfur are needed to make 35 tons of acid?
6. Find the resistance of 300 feet of a wire having .00027 ohms of resistance per foot, if the resistance varies directly as the length.
7. In a circuit of 48 volts a voltmeter register 60 on its scale of 1-100. What is the maximum number of volts the meter can measure?

"PER" PROBLEMS

An important mathematical concept is the idea of "per", as in miles per gallon, population per square mile, etc. Shown below are two possible ways to think of (and solve) "per" problems.

Sample Problem: If 400 pieces of pipe were produced in 8 hours, how many <u>pieces</u> were being produced <u>per hour</u> ?	
Example 1: (Using Division) Think of the "per" as a division sign--so <u>pieces per hour</u> becomes $400 \text{ pieces} \div 8 \text{ hours}$ or $\frac{400 \text{ pieces per hour}}{8 \text{ hours}}$ $400 \div 8 = 50 \text{ pieces per hour}$	Example 2: (Using a Proportion) One way to begin to set up a proportion is to think of <u>pieces per hour</u> as: $\frac{\text{pieces}}{\text{hour}}$ Then set the proportion: $\frac{400 \text{ pieces}}{8 \text{ hours}} = \frac{? \text{ pieces}}{1 \text{ hour}}$ $400 \times 1 = 400 \quad 400 \div 8 = 50 \text{ pieces per hour}$

Below are problems for practice. Be careful. As usual, I'm hoping these will stimulate your brain cells. You can use your calculator for computation, but write down what you did.

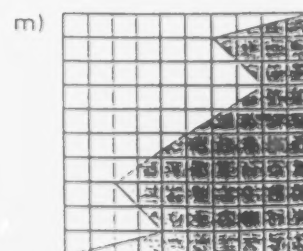
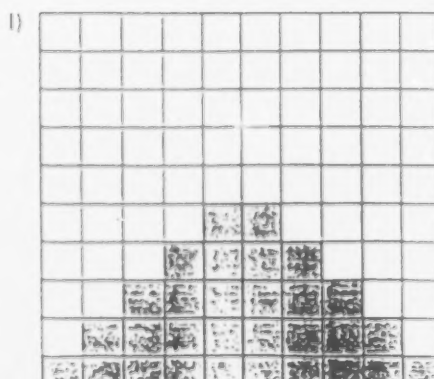
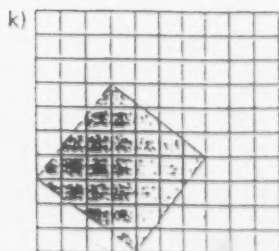
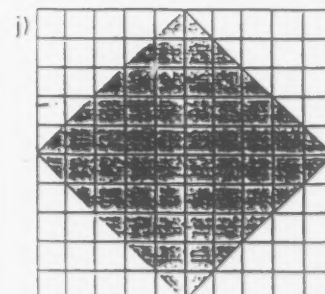
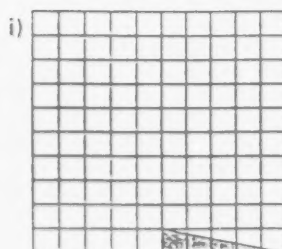
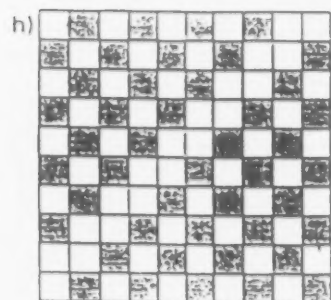
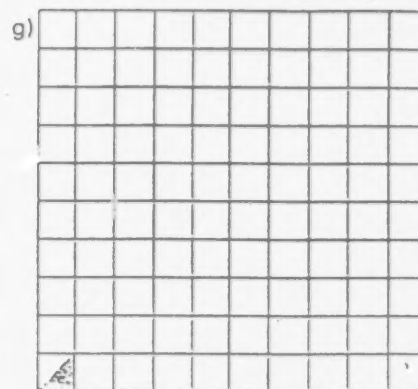
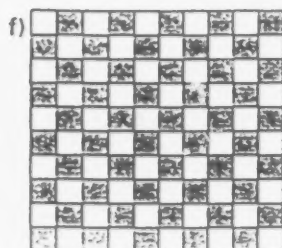
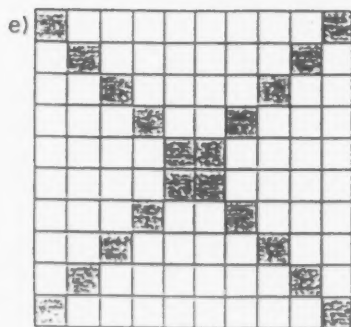
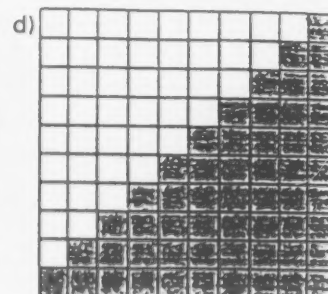
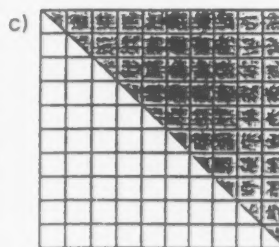
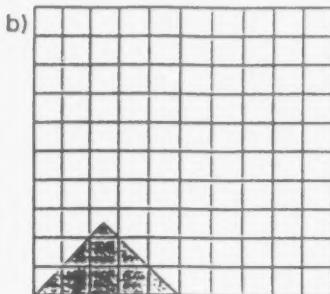
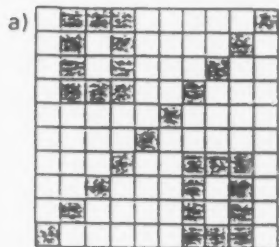
1. A car went 84 miles on 3.5 gallons of gas. How many miles per gallon would this be?
2. A car went 84 miles on 3.5 gallons of gas, how much gas does it use per mile of driving?
3. If Bill usually spends about \$750 a year on clothing for himself. How much money should he budget per month for clothing?
4. In one year, the state of Washington made \$58,800,000 profit from the lottery. In that same year, the population of Washington was 4,300,000. What was the average contribution per person to the lottery?
5. If 3 dozen eggs cost \$2.67, what is the cost per egg?
6. If 3 dozen eggs cost \$2.67, how many eggs can you buy for each dollar?
7. A 12-pound shankless ham contains 16 servings. How many servings per pound is this?
8. If a 12-pound ham contains 16 servings, how much ham is being used per serving?
9. Mary Lou ran 80 yards in 16 seconds? How many yards per second is this?



Focus Student Activity 39.1

NAME _____

DATE _____





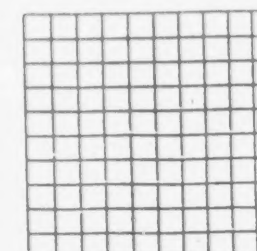
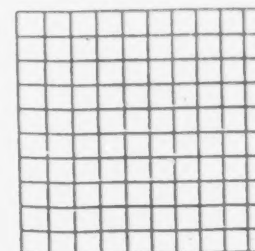
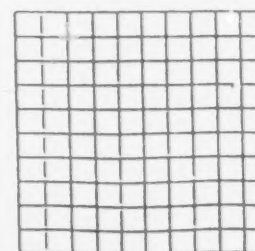
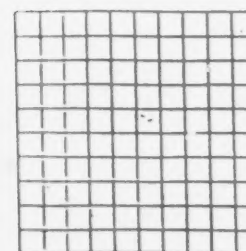
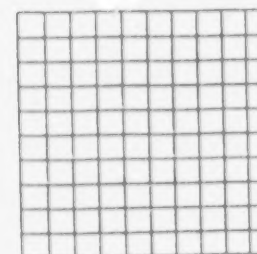
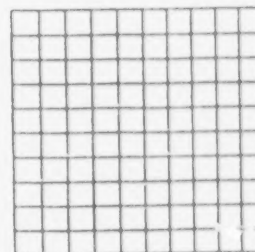
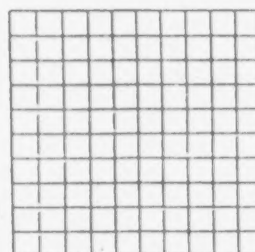
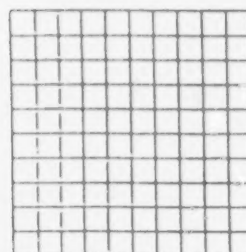
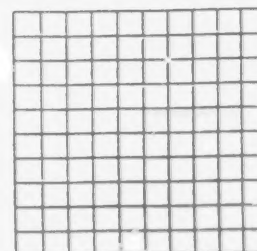
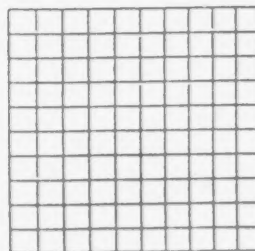
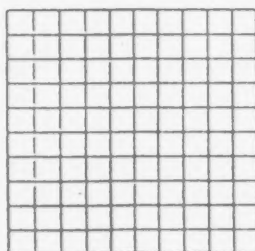
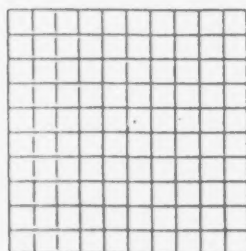
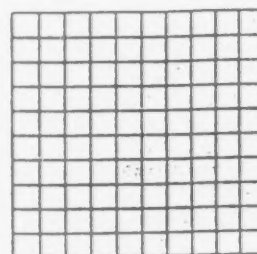
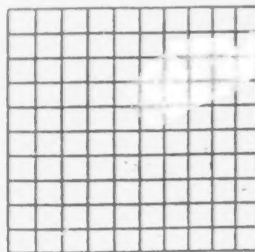
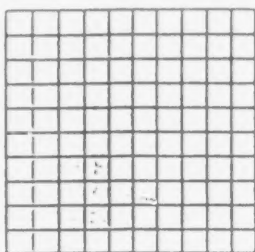
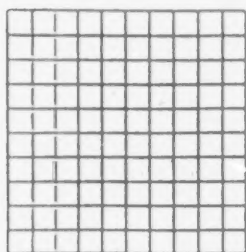
Focus Student Activity 39.2

NAME _____ DATE _____

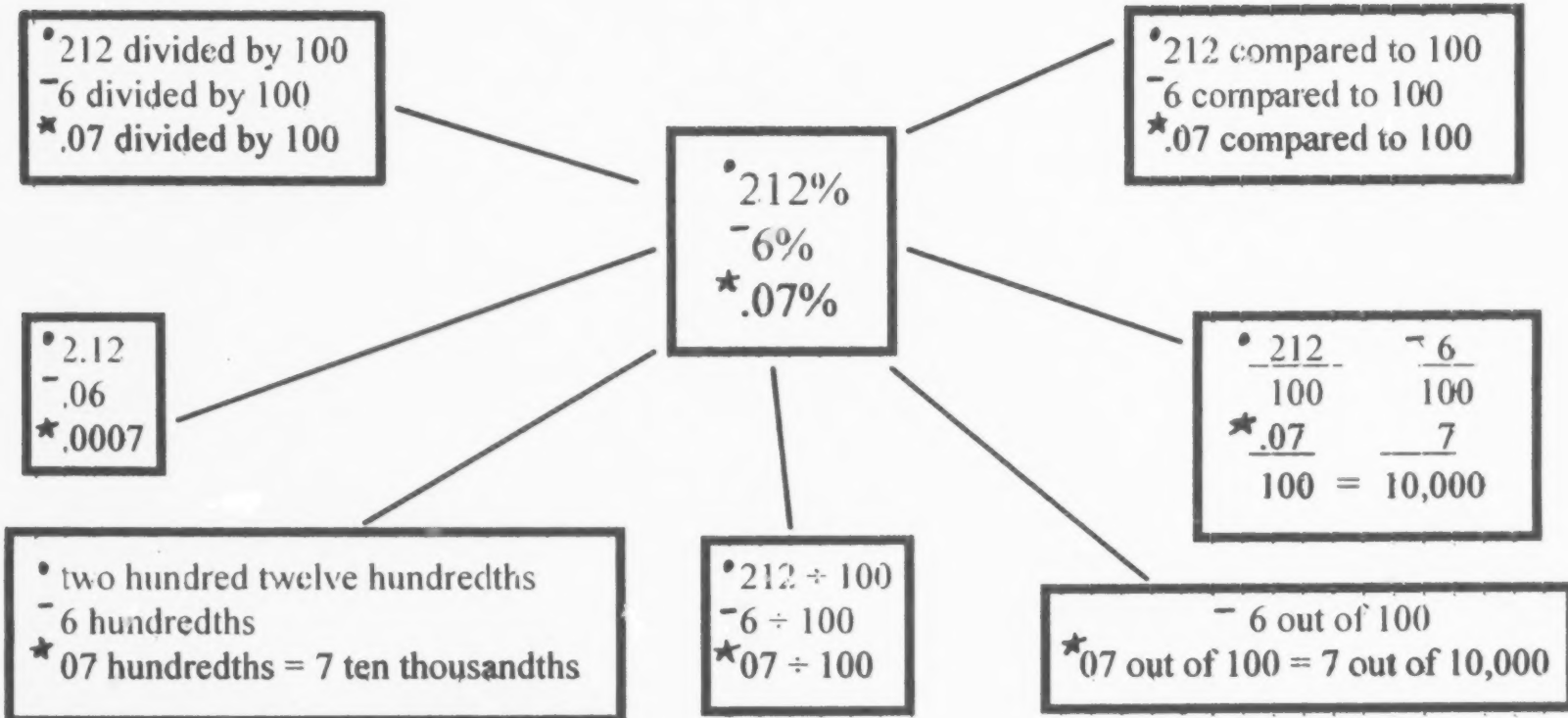
Using the grids below, shade each of the following amounts:

- a) 25% b) $12\frac{1}{2}\%$ c) $\frac{7}{2}\%$ d) .75% e) $\frac{1}{4}\%$
f) 99.9% g) 5.5% h) 137% i) 210%

Label the grids.



PERCENT means PER HUNDRED



1% of 100 = 1
 1% of 200 = 2
 1% of 50 = .5 (1/2)

10% of 100 = 10
 10% of 200 = 20
 10% of 50 = 5
 100% of anything is ALL of it.

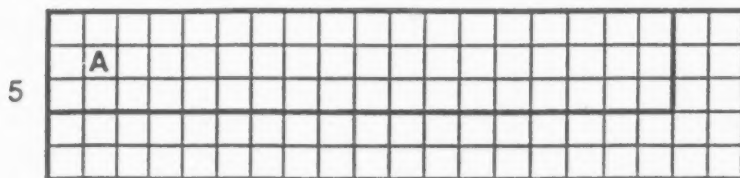
99% of 100 = 99
 99% of 200 = 198
 99% of 50 = 49.5

90

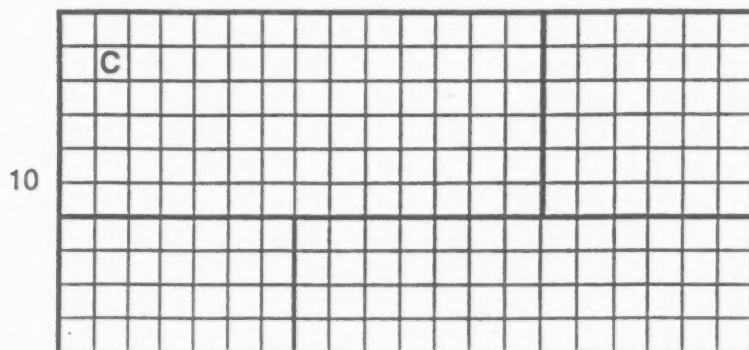
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Name _____

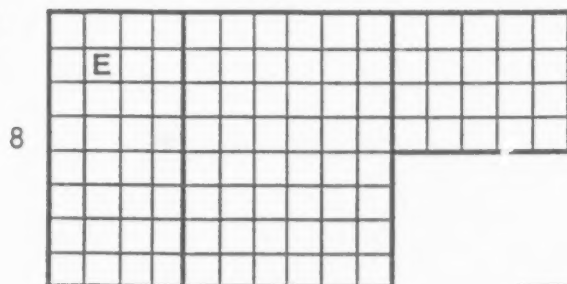
Find what percent the area of each subregion is of the entire region.



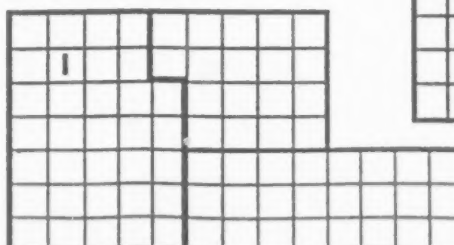
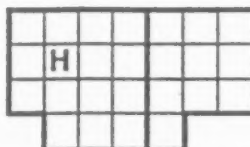
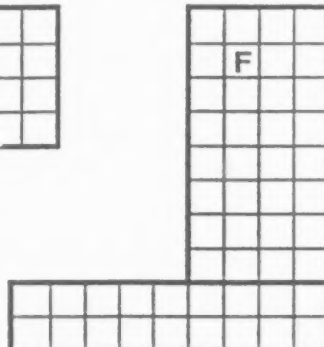
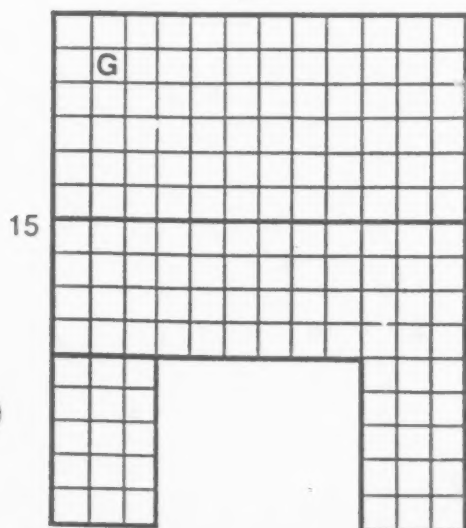
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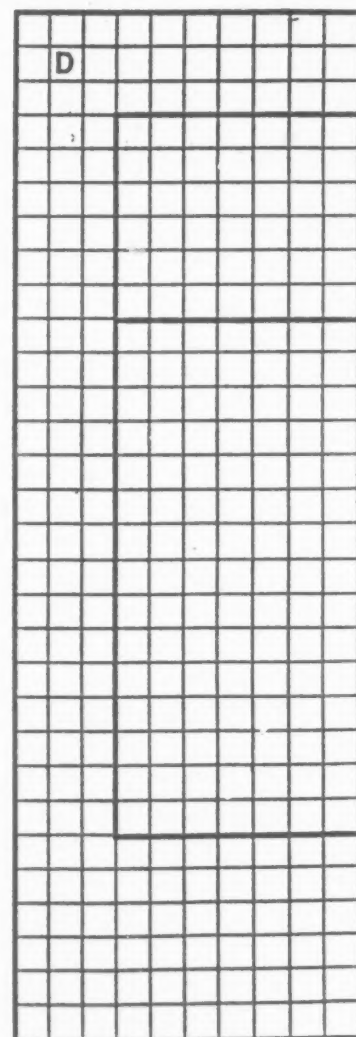
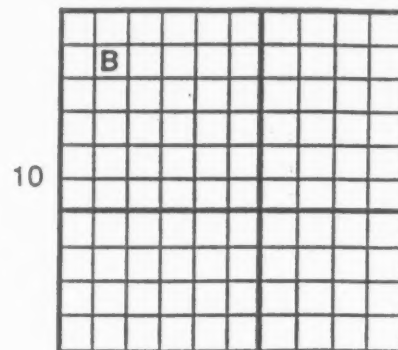
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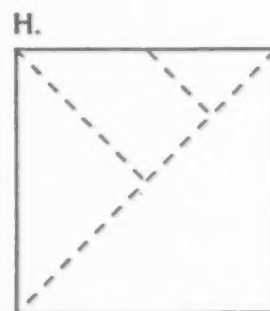
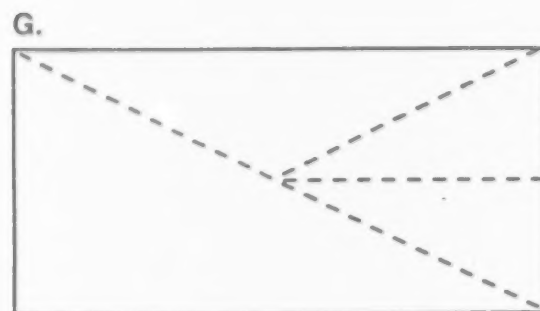
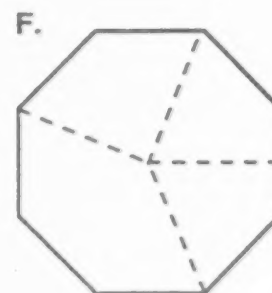
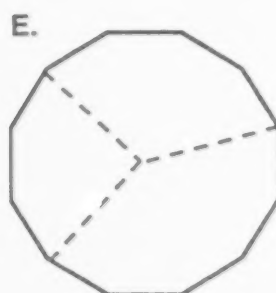
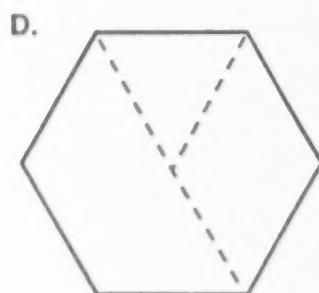
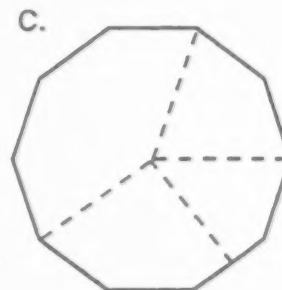
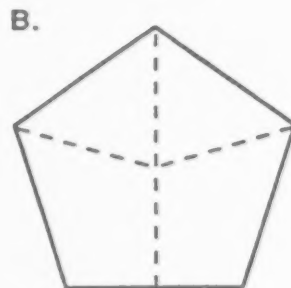
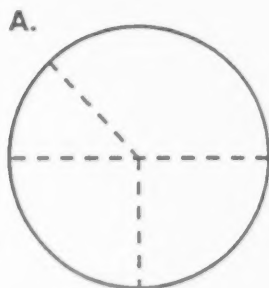


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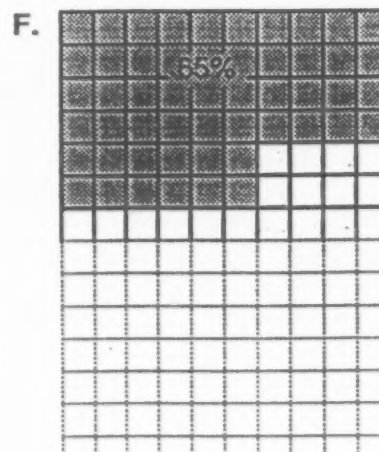
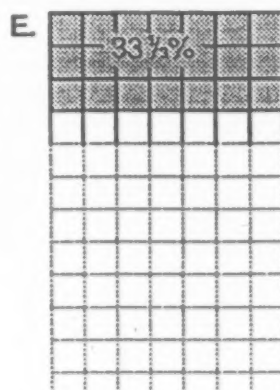
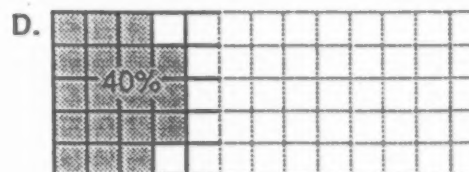
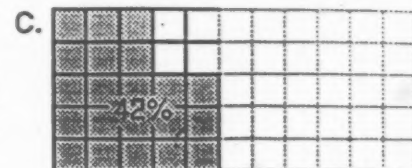
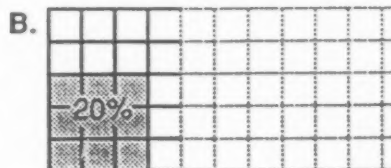
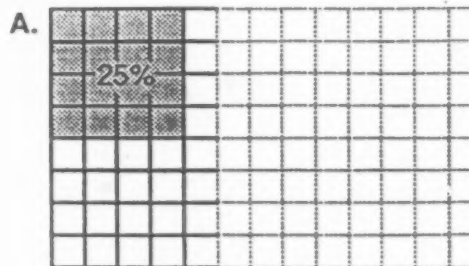
Name _____

What percent is the area of each subregion of the area of the entire region?

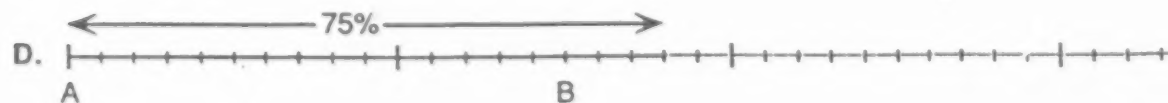
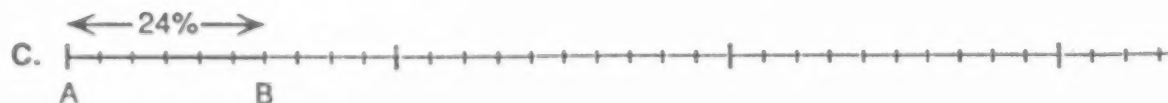
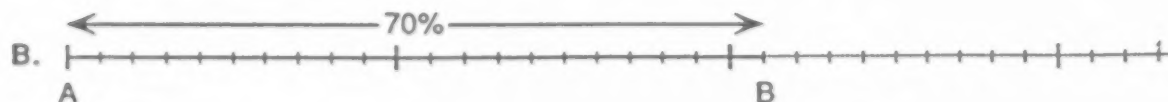
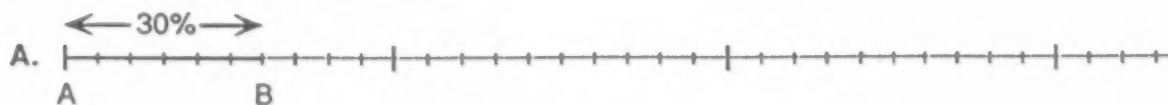


Name _____

I. Complete the rectangle so the shaded region is the given percentage of its area.



II. In each of the following, AB is part of a line segment. Complete the line segment if AB is the given percentage of its length.



Quick and Easy Everyday Percents

100% of something is all of it. 10% is 1/10 of it. 1% is 1/100 of it. 0% is none of it.
50% of something is 1/2 of it. 25% is 1/4 of it. 75% is 3/4 (or 3 times 1/4) of it. 20% is 1/5 of it.
33-1/3% is 1/3 of something. 66-2/3% is 2/3 of it.

You can use these familiar percents to figure out other percents.

_____ % = 25% + 1% _____ % = 10% + 10% + 1% _____ % = 50% - 10%

24% = _____ - _____ 35% = _____ + _____ 9% = _____ - _____ 299% = _____ - _____

Use these ideas to help you find the following percents.

25% of 200 is _____. 1% of 200 is _____. 24% of 200 is _____. 26% of 200 is _____.

50% of 600 is _____. 1% of 600 is _____. 51% of 600 is _____. 49% of 600 is _____.

25% of 160 is _____. 10% of 160 is _____. 35% of 160 is _____. 15% of 160 is _____.

200% of 36 is _____. 25% of 36 is _____. 175% of 36 is _____. 225% of 36 is _____.

100% of 210 is _____. 10% of 210 is _____. 90% of 210 is _____. 110% of 210 is _____.

33-1/3% of 63 is _____. 66-2/3% of 63 is _____. 300% of 7 is _____. 500% of 7 is _____.

SOLVING PERCENT PROBLEMS

Percent problems can be easily solved by first setting up a proportion to represent the problem.
Follow the steps below to set up a proportion for (and then solve) any percent problem.

Step 1: Remember the form a proportion takes:

$$\frac{\square}{\square} = \frac{\square}{\square}$$

Each step below will help you fill in one part of the proportion.

Step 2: Since we are working with a percent problem and since percent means hundredths, your first step should be to put 100 in the bottom of one ratio.

Step 3: Ask "How many percent?" If this information is given, put the number of the percent above the 100 in the first ratio.

Step 4: Ask "What is the total amount we are dealing with?" If this information is given, put this number in the bottom of the second ratio.
(*HINT: In a problem like "3% of ___ is 27" or "What is 15% of 84?", the total is always the number that comes directly after the "of."*)

Step 5: Ask "What part of the total are we dealing with?" If this information is given, put this number on the top of the second ratio.

Step 6: You should now have information in 3 of the 4 parts of the proportion. Solve the proportion and you will have the answer to the percent problem.

SUMMARY:

*To solve a percent problem,
we set up a proportion as follows*

$$\frac{\text{percent}}{100} = \frac{\text{part}}{\text{total (of \#)(base)}}$$

Example: 5% of ___ is 18.

1. Set up a proportion form with 100 in the bottom of one ratio. $\frac{\square}{100} = \frac{\square}{\square}$
2. How many percent? (5)
Put 5 above the 100. $\frac{5}{100} = \frac{\square}{\square}$
3. What is the total? Not given.
Leave the bottom of the second ratio blank. $\frac{5}{100} = \frac{\square}{\square}$
4. What is the part? (18)
Put 18 in the top of the second ratio. $\frac{5}{100} = \frac{18}{\square}$
5. Solve the proportion.
You do this part.

Example: What percent of 125 is 5?
(Follow the steps below to set up the correct proportion in the space provided.)

1. Set up a proportion form with 100 in the bottom of one ratio.
2. How many percent? Not given.
Leave the space above the 100 blank.
3. What is the total? (125)
Put 125 in the bottom of the second ratio.
4. What is the part? (5)
Put 5 in the top of the second ratio.

5. Solve the proportion.

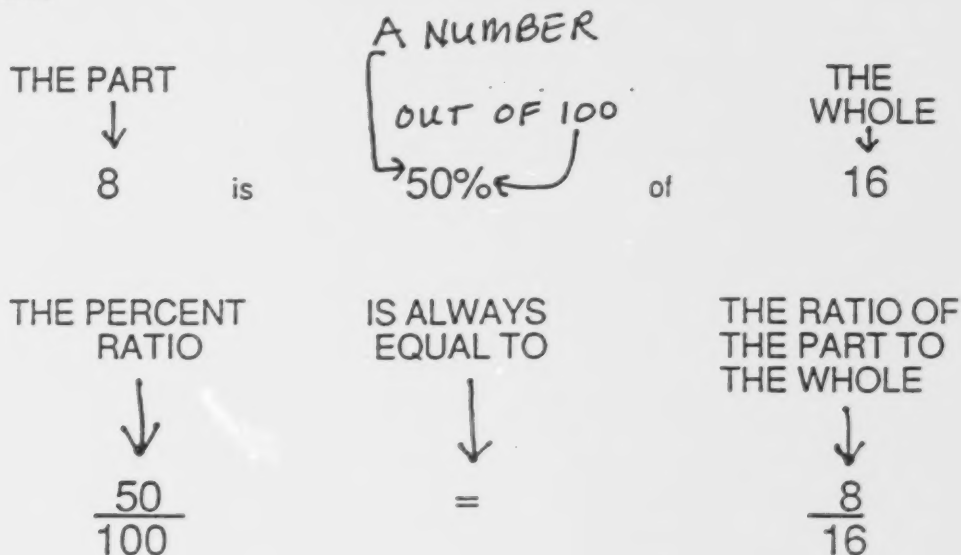
Example: What is 15% of 600?
(Use the following to help you set up the correct proportion.)

1. Put 100 in the bottom of one ratio.
2. How many percent?
3. What is the total?
4. What is the part?
5. Solve.

SAMPLE PROBLEMS WHEN RATIO AND PROPORTION CAN BE USED

PERCENTAGE

Teacher Tips: One of the common uses of proportion is for the solution of percentage problems. One may think of every percentage problem as having four parts:



12 is what % of 16

$$\frac{?}{100} = \frac{12}{16}$$

$$12 \times 100 = 1200$$

$$1200 \div 16 = 75$$

$$\frac{75}{100} = \frac{12}{16}$$

What is 30% of 900?

$$\frac{30}{100} = \frac{?}{900}$$

$$30 \times 900 = 27000$$

$$27000 \div 100 = 270$$

$$\frac{30}{100} = \frac{270}{900}$$

6 is 30% of what?

$$\frac{30}{100} = \frac{6}{?}$$

$$6 \times 100 = 600$$

$$600 \div 30 = 20$$

$$\frac{30}{100} = \frac{6}{20}$$

NAME _____ DATE _____

Percent Problem Chart

The chart below gives the three types of percent problems: (1) Finding the percent of a number (percentage and base given), (2) finding what percent one number is of another (base and percentage given), and (3) finding a number when a percent of it is known (percent and percentage given). The first problem has been done as an example. Try to find the other missing numbers. Time is limited to 20 minutes.

No.	Percent	Base	Percentage
1.	18%	46	8.28
2.		12	6
3.	36%		18
4.	6%	24	
5.	25%		6
6.		8	7
7.	3%	5,000	
8.		54	45
9.	6%		12
10.	83 $\frac{1}{3}$ %	582	
11.	120%		108
12.		38	38
13.	40%		12.6
14.		935	561
15.	4 $\frac{1}{2}$ %	624	
16.	60 $\frac{3}{4}$ %	940	
17.		\$34.00	\$1.70
18.	2 $\frac{1}{2}$ %		2
19.	187 $\frac{1}{2}$ %		210
20.		10 $\frac{1}{2}$	3 $\frac{1}{2}$
21.	.5%		8
22.		25	13
23.	6%	\$14.50	
24.	$\frac{3}{4}$ %	54	
25.		10	18



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Scores:

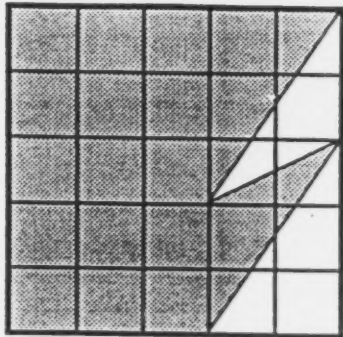
0-4, Finished Last
 9-12, Wait Till Next Year
 18-22, Super Bowler

5-8, Future Looks Bad
 13-17, In the Playoffs
 23-25, Super Bowl Champ

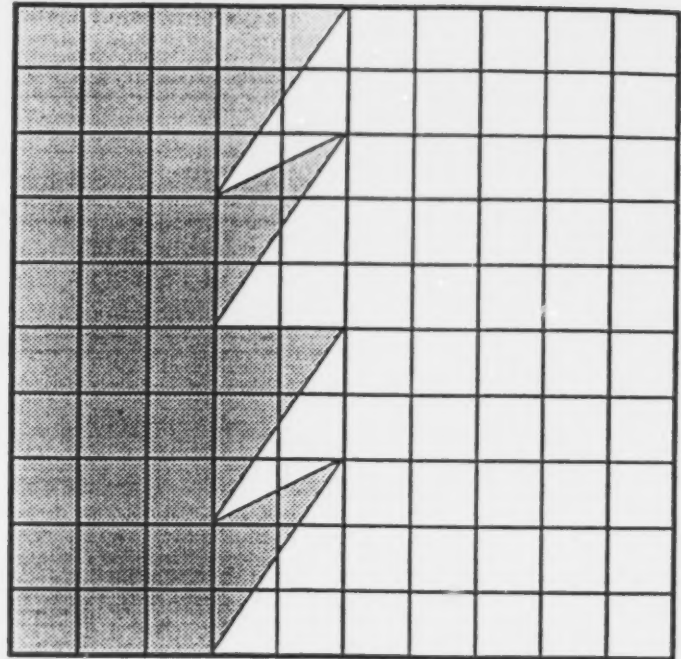


Have Fun with Math

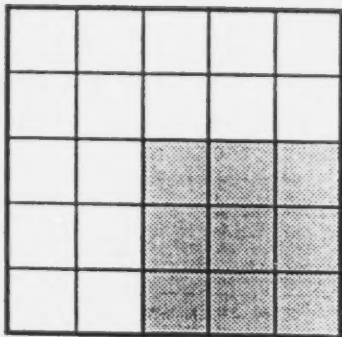
Figures from Activity 1
(Math & the Mind's Eye, Unit VII)



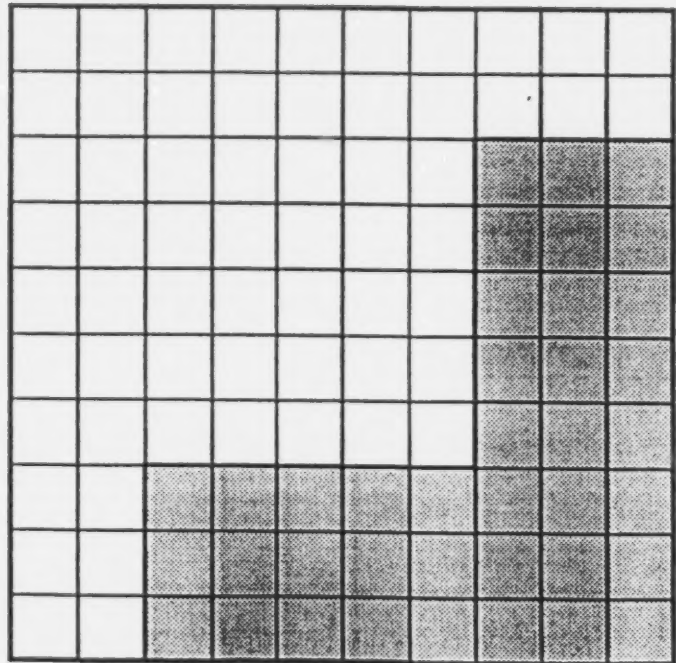
A



B



C



D

PERCENT COMPUTATION REVIEW

Change the following percents to decimals.

1. 20% _____ 2. 100% _____ 3. 82.5% _____ 4. 5.9% _____ 5. 75% _____

Change the following percents to fractions.

6. 76% _____ 7. 5% _____ 8. $33\frac{1}{3}\%$ _____ 9. $12\frac{1}{2}\%$ _____

Change the following decimals to percents.

10. .73 _____ 11. .9 _____ 12. .003 _____ 13. 5.3 _____ 14. .08- $\frac{1}{3}$ _____

Change the following fractions to percents.

15. $\frac{1}{5}$ _____ 16. $\frac{3}{8}$ _____ 17. $\frac{5}{12}$ _____ 18. $\frac{4}{3}$ _____ 19. $\frac{6}{7}$ _____

Find the following percentages.

20. 5% of 340 _____ 21. 25% of 280 _____ 22. $33\frac{1}{3}\%$ of 540 _____

23. 110% of 448 _____

Find the following percents.

24. 8 is what percent of 16? _____ 25. 7 is what percent of 35? _____

26. 250 is what percent of 75? _____

Find the whole amount in the following.

27. 50% of what number is 6? _____ 28. $66\frac{2}{3}\%$ of what number is 82? _____

29. 150% of what number is 240? _____

Find the new amount in each of the following.

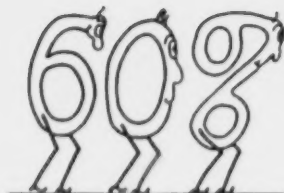
30. 300 increased by 25% = _____ 31. 900 decreased by 15% = _____

Find the percent of increase or decrease.

32. From 6 to 9 is an increase of _____ %. 33. From 120 to 80 is a decrease of _____ %

Mystic Square with Percent

Solve each of the 16 problems in the list below. Then place each answer in the cell of the square with the same number as the problem. When all the answers have been placed in the square, find the sums of the rows of answers, columns of answers, and diagonal of answers. If the answers are all correct, all the sums will be the same.



Sum

Sum				
1.	2.	3.	4.	
5.	6.	7.	8.	
9.	10.	11.	12.	
13.	14.	15.	16.	



- Express 80% as a decimal number.
- Numerator when 95% is expressed as a fraction and reduced to its simplest form.
- Find 15% of 118.
- 9.4 is what percent of 200, expressed as a decimal?
- 7.55 is 50% of what number?
- Express 730% as a decimal number.
- Determine $33\frac{1}{3}\%$ of 25.8 and round to the nearest tenth.
- Percent, expressed as a decimal, when 126 is divided by 1,125.
- $12\frac{1}{2}\%$ is 125% of what number?
- Find the percent, expressed as a decimal, that 14.7 is of 117.6.
- What number is $2\frac{1}{2}\%$ of 552?
- Find $1\frac{1}{4}\%$ of 480.
- 225% of what number is 36.9?
- What number is 32% less than 5?
- Find $\frac{1}{2}\%$ of 420.
- What percent gain, expressed as a decimal, is an increase from \$180 to \$216.54?

WORK-RELATED MATH REVIEW (A)

- 1) Safety cost is figured as cost per hour worked--that is, the total cost of safety incidents divided by the total hours worked. Total hours worked per month is approximately 9,000. An incident happened in February that cost \$420, figure the safety cost for the first 2 months of the year if this is the only safety incident so far

- 2) Safety cost is figured as cost per hour worked--that is, the total cost of safety incidents divided by the total hours worked. The total hours worked per month is approximately 9,000. In February an incident happened that cost \$420 and in early March there was another incident which cost \$150. If it is now the end of March, figure the safety cost for the year so far.

- 3) This week you worked 42 hours (6 of which were overtime). The basic pay rate per hour is \$14.15; the shift differential for your shift is \$.45 per hour. Overtime is paid at time and a half (based on your full hourly rate). Figure your gross pay for the week.

- 4) This week you worked 42 hours (6 of which were overtime). Your basic hourly pay rate is \$12.50; the shift differential for your shift is \$.45 per hour. Overtime is paid at time and a half (based on your full hourly rate--basic rate plus shift differential). Figure your gross pay for the week.

- 5) If the blending capacity is 2700 lbs. per hour for each of the 3 blenders and they are run for 24 hours, will the plant's production of 205,000 lbs. for the same 24 hours be more or less than the blender capacity? By how much?

- 6) If the blending capacity is 2700 pounds per hour for each of the 3 blenders and they are run for 24 hours, will the plant's production of 198,000 pounds for the same 24 hours be more or less than the blender capacity? By how much?

WORK-RELATED MATH REVIEW (C)

- 14) You are running an order of 8,600 ft. and are told to change to a different crate quantity when the run is 60% complete. At how many feet would you make the change? If these were 20-footers, how many pieces would you have at this point?
- 15) A customer requested that 60% of their current order of 9000 feet of pipe should be 6" pipe. How many feet of 6" pipe would this be?
- 16) This month 7,940 pounds of pipe were rejected out of 5,292,000 pounds produced. Calculate the Q.A. reject percent for the month.
- 17) This month 4,234 pounds of pipe were rejected out of 5,292,000 pounds produced. Calculate the Q. A. reject percent for the month.
- 18) You are producing 13-foot pipe that weighs 3.921 pounds per foot. If, out of a total production of 13,200 pounds, 15 pieces were thrown into the grinder, what was the percent of regrind generated?
- 19) You are producing 13-foot pipe that weighs 3.921 pounds per foot. If, out of a total production of 13,200 pounds, 13 pieces were thrown into the grinder, what was the percent of regrind generated?
- 20) If you have a product running at 1775 pounds per hour and the standard rate for that product is 1800 pounds per hour, what is your percent of standard?
- 21) If you have a product running at 1872 lbs. per hour and the standard rate for that product is 1800 lbs. per hour, what is your percent of standard?

Work-Related Math Review (D)

1. Below are some figures for pipe production during one week. Use your estimation and reasoning powers to decide on which day the most usable pipe was produced and on which day the plant had the best "percent of reject" figures. Be prepared to explain your reasoning.

Monday	150,000 pounds produced	505 pounds rejected
Tuesday	147,550 pounds produced	470 pounds rejected
Wednesday	139,800 pounds produced	120 pounds rejected
Thursday	151,050 pounds produced	1095 pound rejected
Friday	145,100 pounds produced	365 pounds rejected
Saturday	142,230 pounds produced	290 pounds rejected
Sunday	146,230 pounds produced	920 pounds rejected

2. Now, do the calculations to determine the actual pounds of pipe produced for the week and the actual percent of rejected pipe for each day (to the nearest thousandth of a percent). How close were your estimates from #1?

3. Compare the daily pounds produced and percent of reject figures that you produced in #2. Do you see any patterns, trends, relationships? If this was all the data you had, are there any speculations you might make about pipe production? (Remember that this is all imaginary data, so don't depend on any conclusions you might draw.)

4. You want to speed up the extruder speed from 1400 to 1750 rpm. The current puller speed is 475. Use 2 different methods to find the new puller speed.

5. You are operating line #3 at 1000 pounds per hour and the standard rate for that line is 1250 pounds per hour. At what percent of standard are you operating? If the extruder rpm is 1700 now, what theoretical rpm would it take to get to standard rate?

Work-Related Math Review (E)

For the following questions, you can use a calculator for all calculations, but, for each one please show clearly on another sheet of paper the calculations you did and the answer you got.

1. Today you have produced 1690 feet of pipe with a total weight of 7551 pounds. What is the weight per foot of this pipe? (You can round the answers you write to 2 decimal places--hundredths--but please keep the process continuous in the calculator until you've finished question #4.) How much would 100 feet of this pipe weigh?
2. If the perfect weight of 100 feet of this pipe is 434.99 pounds, what is the ratio of weight of 100 feet of the pipe produced today (answer from #1) to the weight of 100 feet of perfect pipe?
3. Change the ratio you determined in #2 to a percent.
4. Hopefully, you arrived at an answer of 102.716%--which means that the pipe you produced weighs 102.716% of the weight of perfect pipe. In that case, by what percent is your pipe overweight?
5. You have 2200 feet of 4" 160# IPS pipe with a total weight of 3245 pounds. What is the weight per foot of this pipe? (Again, you can round the answers you write to hundredths, but please keep the process continuous in the calculator until you finish question #7.) How much would 100 feet of this pipe weigh?
6. If the perfect weight of 100 feet of this pipe is 148.593 pounds, write the ratio of the weight of the pipe in question #5 to the weight of the perfect pipe. Now, express that ratio as a percent. What is your percent of overweight?
7. Let's look at this a little differently. What is the difference between the weight of 100 feet of your "real" pipe (2200 feet weighing 3245 pounds) and the weight of 100 feet of perfect pipe (given in question #6)? (Please re-calculate this information so you'll have continuous calculation process.) Write the ratio comparing this difference to the perfect 100 foot measurement(Magic Number); then change this ratio to a percent. (Compare this answer with your answer in #6.)

NOTE: If your real pipe is actually overweight, your percent of overweight will be a positive percent. If your real pipe is underweight, your percent of overweight will be a negative percent--indicating that the pipe is not actually overweight.

8. Try to explain what was different in the process used in #7 from that used in #5 & 6.
9. Note the calculations of pipe overweight in problems #1-4 and #5-7 could also have been figured using the formula:

$$\text{Total Weight} \div \text{Total Footage} \div \text{Magic Number} \times 10,000 - 100.$$

Can you spot how the calculations in this formula match with those we did in the earlier problems? Now, use this formula and the data from problems #1-4 and #5-7 to figure the necessary overweights.

Math Review Posttest

For each of the following questions, carefully recapture what you have learned and transfer it to the paper. You may use a calculator on Questions 3, 4, and 5.

1) Circle the four of the following that are all equal to each other.

800% 80 .08 8/10 8% 8/100 .008 2/25 .08%

2) Circle the number below that is the most reasonable answer to: $31.89 \times .97$
(Please note that none of the possible answers given are the exact answer.)

309,000 .309 39 3090 30.9 3.93 .0039

Explain your choice. _____

3) The Magic Number is defined as the weight per 100 feet of perfect pipe. Figure the Magic Number of 18" 100# PIP if 2800 perfect feet of it weights 47,111 pounds.

MATH COMPETENCIES

SKILL	LEVEL	COMMENTS
read/use chart/graph		
estimate		
communicate information		
show steps		
interpret/follow directions		
sequencing multi-step prbs.		
multiply whole #		
divide whole #		
fraction sense		
decimal sense		
add decimals		
subtract decimals		
multiply decimals		
divide decimals		
decimal to fraction		
fraction to decimal		
find average		
percent sense		
ratio/proportions		
solve percent w/proportion		
solve percent other ways		
problem-solve chang dimen		
problem-solve w/whole #		
problem-solve w/decimal		
problem-solve w/fractions		
problem-solve w/percents		
area/volume sense		
work-specific understanding		

Scale for above:

NA - Not applicable; we didn't really work on this

1 - Got a chance to see it done

2 - Practiced a little

3 - Beginning to understand

4 - Pretty clear, but sometimes slips away

5 - GOT IT

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Course Title: Basic Math

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"Equal Fractions, Decimals, and Percents," Have Fun with Math

"Estimation Quotients" Calculate!

"Finding Averages" Mary Mayfield, 1995.

"Focus Master A (Situations)" "Modeling Percent," Visual Mathematics, Course 1, The Math Learning Center, Salem, OR, 1994. (Used with Permission)

"Focus Master B (10 x 10 grid paper)" Visual Mathematics, Course 1

"Focus Student Activity 39.1," Visual Mathematics, Course 1

"Focus Student Activity 39.2," Visual Mathematics, Course 1

"General Multiplication/Division Word Problems (Teacher Tips page and worksheet page)," pp. 6-7, GED Priority Math, Mary Ann Schmidt, Massachusetts Department of Education, 1989. (Used with Permission)

"Going in Circles" Problem Solving in Mathematics, Lane County Math Project, 4th Grade Book, 1983. (Public Domain)

"Ground Rules" Mary Mayfield, 1996.

"Hit the Target," Calculate!

"Lineup with Decimals," Have Fun with Math, Louis Grant Brandes, J. Weston Walch, Portland, Maine 1988.

"Lineup with Fractions, Decimals, and Percents," Have Fun with Math.

"Math Anxiety Bill of Rights" Mary Mayfield, 1996.

"Math Competencies" Mary Mayfield, 1997.

"Math Practice Log" Mary Mayfield, 1996.

"Math Review Posttest" Mary Mayfield, 1995.

"Math Training Preview" Mary Mayfield, 1996.

"Mystic Square with Percent," Have Fun with Math.

"Number Patterns, Problem-Solving, and Review" Mary Mayfield, 1996.

Course Title: Basic Math

- "*Number-Line Decimals*," Problem Solving in Mathematics, Lane County Mathematics Project, Grade 5, p. 197, 1983, Dale Seymour Publications. (Used with Permission)
- "*Open Calculation*" Mary Mayfield, 1996.
- "*Per-Problems*," Mary Mayfield, 1997.
- "*Percent Computation Review*," Mary Mayfield, 1997.
- "*Percent Means Per Hundred*" Mary Mayfield, 1996.
- "*Percent Problem Chart*," Have Fun with Math
- "*Percentage*" ("*Teacher Tips*" page and worksheet), "*Ratio and Proportion*", GED Priority Math.
- "*Placing the Point 1*," Calculate!
- "*Placing the Point 2*," Calculate!
- "*Practice with Ratios and Proportions*," Mary Mayfield, 1996.
- "*Problem Solving Possibilities*," Mary Mayfield, 1996.
- "*Problems Stated as Ratios* (Teacher Tips page and worksheet page)," GED Priority Math, pp. 4-5.
- "*Quick and Easy Everyday Percents*," Mary Mayfield, 1996.
- "*Ratio and Proportion: Unit 4*," pages 128-149, Decimals, Fractions, Ratios, and Percents (part of the series called SMART SOLUTIONS: Skills, Problem Solving, Tools, and Applications), 1995, Ellen Carley Frechette, New Readers Press, Publishing Division of Laubach Literacy International, Box 131, Syracuse, New York 13210-0131.
- "*Rules for Computing with Decimals (2): Subtraction*," Mary Mayfield, 1997.
- "*Rules for Computing with Decimals (3): Multiplication*," Mary Mayfield, 1997.
- "*Rules for Computing with Decimals (4): Division*," Mary Mayfield, 1997.
- "*Rules for Computing with Decimals(1): Addition*," Mary Mayfield, 1997.
- "*Secret Message Using Decimals*," Have Fun with Math.
- "*Similar Shapes* (Teacher Tips page and worksheet page), " pp. 12-13, GED Priority Math.
- "*Solving a Percent Equation*" Decimals, Fractions, Ratios, and Percents, pp. 166,167.
- "*Solving Percent Problems*" Mary Mayfield, 1978, (revised 1997)

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"The Percent Equation" Decimals, Fractions, Ratios, and Percents, pp. 164, 165.

"Using Proportions: Bicycle Gears," copied from old discarded elementary math books.

"Using Proportions: Estimation of Fish Population," copied from old discarded elementary math books.

"Using Proportions: Model Airplanes" copied from old discarded elementary math books.

"Which Answer is Reasonable?" Mary Mayfield, 1997.

"Which of These Is .5%?," Mary Mayfield, 1996.

"Word Problems with Decimals," Mary Mayfield, 1996.

"Word Problems with Percents and Proportions," Mary Mayfield, 1996.

"Work-Related Math A, B, C, D, E," (B not included) Mary Mayfield, 1996-97.

Figures from Math and the Mind's Eye, Unit VII, Activity 1 ("Introduction to Percentages") and Activity 2 ("Fractions, Decimals & Percents")

Ratio and Proportion, Connie Eichhorn, Cambridge Adult Education, Power Math Series, pp. 53 and 56.

Appropriate worksheets or pages from workbooks for practice of addition, subtraction, multiplication, and division--singly and mixed (The instructor used appropriate pages from Contemporary's Math GED book and from Contemporary's Decimals booklet as well as some worksheets, but there are many possibilities for appropriate practice.

Workplace Training Project

Course Title: Fractions and Measurement in the Workplace

Course Description:

Course reviewed fractional concepts (reading, writing, comparing fractions, etc.) and computation with fractions. It gave participants practice in solving word problems and in solving practical problems from the workplace. To facilitate understanding of workplace problems, participants had some instruction and practice in reading blueprints and diagrams. There was also some instruction and practice in using a tape measure and other measuring tools. Throughout the course, there was an emphasis on problem-solving.

Student Outcome Objectives

Participants will demonstrate the ability to:

1. Understand fractions well enough to compare them both by reasoning and by using common denominators.
2. Compute with any combination of fractions, whole numbers and mixed numbers.
3. Work with a variety of word problems until participants understand when to add, subtract, multiply or divide.
4. Solve workplace problems that involve understanding a variety of workplace situations, being able to read a blueprint/diagram, and applying the appropriate computations.
5. Read a tape measure and use a tape to measure a variety of materials of different sizes and shapes with an expected accuracy to $\frac{1}{32}$ of an inch.

Course Title: Fractions and Measurement in the Workplace

Student Population:

The class consisted of 13 employees of a secondary wood products manufacturing plant. These participants worked throughout the various departments of the plant and included one supervisor ("lead.") All participants took the class voluntarily. All of these employees need to use fractions and measurements daily in their work.

A second class at the same plant had 4 employees and included no supervisors.

Class Environment:

In the beginning, the class met in the employee break room because it was the only available space that seemed big enough. Unfortunately, its shape and furnishings did not lend themselves comfortably to group instruction. The picnic tables and benches were awkward in an instructional setting. The room has an echo and, being next to the plant, was noisy enough that the instructor really needed to project her voice. Even so, she wasn't always sure the participants were able to hear her. Several class days the temperature was in the 90's and this room was very hot. The plant supplied a portable fan that helped with cooling but added to the noise level.

A couple of participants were reluctant to be in class, and sometimes they resisted being part of group activities. The nature of the space made it easier for them to withdraw from the rest of the group. This contributed to the class being more fragmented than was desirable.

Later, the class met in the smaller, but air-conditioned conference room. This was an improvement, but the room was too small for the instructor to move around enough to watch and/or help all participants as they worked (a significant disadvantage, at least for this instructor).

The second class originally met in the break room, but moved outside under some trees to a picnic table about half the time because of the heat. One participant (the one who had the highest skill level) felt forced to attend the class, and that affected his attitude and willingness to participate. Another participant (the one with the lowest skill level) had very sporadic attendance. Attendance ranged from one to four participants. Because the class was much smaller and attendance irregular, it required a very different approach from the first class.

Instructional Techniques: ✓

☐ Multi-media	✓ Small group	✓ Cooperative learning
✓ Lecture	☐ Computer-aided	✓ Assignments
✓ Individualized instruction	☐ Simulation	✓ Peer tutoring
☐ Role-playing	☐ Project	✓ In-class practice

Fractions and Measurement in the Workplace

Syllabus of Course Activities:

I. Introductory material

- Grant paperwork and pretest
- Review of objectives to be achieved in class and what they mean
- Add any additional objectives participants want covered in the class
- Preparation of individualized learning plan

II. Working with tape measures and measurement

III. Making sense of fractions

- Definitions and overview of fractions and what fractions mean
- Activities with whole things
- Working with equivalent fractions

IV. Computations with fractions

V. Workplace production problems using fractions

- Word problems reflecting fraction use in practical, everyday situations and involving skills that would be applicable to the workplace.
- Word problems reflecting fraction use in actual work place situations
- Computations based on production sheets (work orders giving process instructions and dimensional requirements)

VI. Concluding activities including grant paperwork, post-test and feedback

Course Title: Fractions and Measurement in the Workplace

Activity I: *Introductory Activities*

1. Grant paperwork and pretest
2. Review and discussion of class objectives
3. Learning style activity and discussion
4. Add any additional objectives participants want covered in the class
5. Preparation of individualized learning plan

Resources and Materials Needed:

- Grant paperwork.
- "Diagnostic Test" (This pretest is similar to "Fraction Posttest, Part I," which is included with this curriculum.) Mary Mayfield, Instructor, Lane Community College, 1995.
- Individual Learning Plan forms with class objectives for participants to review.

Descriptions of Instruction:

- Instructor leads the class in discussion learning differences after an introductory learning styles activity.
- Participants fill out grant paperwork.
- Instructor reviews with participants the course objectives and gives them an opportunity to add other objectives.
- Participants fill out Individual Learning Plan forms.
- Participants take pretest.

Performance Assessment Methods and Results:

- Completion of forms and individualized learning plans. The participants need to know what will be covered and that this class will meet their needs. Reviewing the objectives and creating an ILP achieves that goal with the participants.

Course Title: Fractions and Measurement in the Workplace

Activity II: *Working with Tape Measures and Measurement*

1. Using tape measures accurately in the workplace

Resources and Materials Needed:

- Tape measure, objects to measure.
- # "Measurement Practice - 1" handout. David Shellabarger, Instructor, Lane Community College, 1995.
- # "Measurement Practice - 2" handout. David Shellabarger, 1995.
- # "Giant Inches", scaled in eighths and sixteenths (p.1), and thirty-seconds (p.2), Mary Mayfield, Instructor, Lane Community College, 1995.
- # "Developing Fraction Sense - Ruler Exercise" David Shellabarger, 1995.
- # "Tape Measure Math" Mary Mayfield, 1995.

Description of Instruction:

1. The class did a fair amount of measuring. Participants used the worksheet "Measurement Practice - 1" which depicts a variety of objects found in the workplace. The worksheet is a good first step in learning to measure. For more practice, instructor used the worksheet "Measurement Practice - 2". This worksheet has a ruler pictured on it (not to scale) and has participants read off measurements of lines to the nearest $\frac{1}{8}$ " and to the nearest $\frac{1}{16}$ ". It also gives participants some practice estimating the lengths of lines before measuring them. The class decided to what accuracy to measure the objects. Surprisingly (to the instructor), people did not come up with the same answers!
2. The instructor used "Giant Inches" (as overhead and as worksheets) to discuss and practice naming the fractional parts of an inch as shown on a tape.
3. "Developing Fraction Sense - Ruler Exercise" offers some measurement practice and uses measurements to introduce computation with fractional numbers. The worksheet gives problems with addition, subtraction, and multiplication (but not division).
4. The instructor and participants collected several different sizes of pieces of wood from work areas, and participants measured them for practice. The instructor offered some reminders about measuring such as:
 - Measure from the 1" mark (and subtract the additional inch) instead of from the beginning of the tape to avoid error caused by fastener at the end of the tape.
 - Measure from 1' mark (and subtract that 1' from your measurement) instead of from the 1" mark because you're more likely to notice an error of 1' if you forget to subtract that additional foot.

Course Title: Fractions and Measurement in the Workplace

5. One of the things participants learned to do mathematically was to center a hole. Some people also did that on their tapes by putting, for instance, the 5" mark approximately where the hole would go, "balancing" their tape until they had the same amount on either side of the 5" mark, then marking the hole at the 5" mark. This actually proved to be quicker for many people than computing it. Those who used this method demonstrated it to others who then tried it out to see how it worked.
6. The instructor emphasized to the participants that using the ruler was still math. They had to be able to read the scale in order to make the results come out accurately. People often have a very limited view of math, not understanding that math really includes a lot of conceptual, logical, and visual things we use every day without realizing it.
7. Participants worked individually and/or in pairs on "*Tape Measure Math*" using their number sense and their tape measure as needed.

Performance Assessment Methods and Results:

- The instructor assessed performance based on the accuracy of the results of participant's measurements. She looked for repeatability, and similarity of conclusions among participants.

Comments:

- This seems like a skill participants would already know, but the review and work on it increased the skill, accuracy, rapidity, and confidence with which participants could measure. A number of participants really didn't understand what the fraction lines meant on the tape and how to read it, so this practice and work with the "*Giant Inches*" helped them.
- "*Tape Measure Math*" was designed to give participants a) more practice with their tape measure, b) more experience working with tape measure fractions, and c) a visual, hands-on sense of some fraction operations.
- "*Developing Fraction Sense - Ruler Exercise*" was useful. However, instead of following the directions (that is, make a drawing and try to get a mental picture and understanding of computing with fractions), many participants tried to use what they remembered (imperfectly) about common denominators or other computational techniques, and then became confused. The instructor had to ask them to suspend for the moment what they remembered and just work with the visual. Following directions was not always a strong skill, so this provided practice on doing only what the directions said.

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Activity III: *Making Sense of Fractions*

1. Definitions and overview of fractions and how they are used at this workplace
2. Activities to demonstrate and stimulate discussion about how we limit or narrow our thinking about a problem
3. Activities with whole things
4. Practice identifying and naming fractions, recognizing the whole and the part
5. Work with equivalent fractions
6. Comparison of fractions

Resources and Materials Needed:

- # "A Whole Thing ①" from Aftermath I, p. 79, Seymour and others, Creative Publications, 1975
- Worksheets from Developing Mathematical Thinking: A Complete Unit on Fractions with Blackline Masters and Teacher Lesson Plans, Jay Greenwood, 1986.
 - # "Sequence A - Lesson 4" (p. 17)
 - # "Sequence A - Lesson 6" (p. 23) (as revised for tape measure fractions by Mary Mayfield)
 - # "Sequence A - Lesson 2" (pp. 10-12)
 - # "Sequence C - Lesson 2" (p. 75)
 - # "Sequence D - Lesson 2" (p. 91)
 - # "Sequence D - Lesson 6: Why do you think so?" (p. 99)
 - # "Sequence G - Lesson 1" (p. 141)
 - # "Sequence G - Lesson 2" (p. 145)
- # "Jumping Floozie" p. 79. Problem Solving in Mathematics, Lane County Mathematics Project, Grade 7, Dale Seymour Publications, Palo Alto, CA, 1983.
- # "A Whole Thing ②" from Aftermath IV, p. 52, Seymour and others, Creative Publications, 1975.
- # "A Whole Thing ③" from Aftermath I, p. 23, Seymour and others, Creative Publications, 1975.
- # "Equivalent Fractions" Mary Mayfield, 1996.
- # "Number Line" with tape measure fractions, Mary Mayfield, 1996.
- Fractions, (Math Exercises series), Contemporary Books, Inc., 1993.
- # "Thinking about Fractions" Mary Mayfield, 1996.

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- # "Line up with Fractions", p. 32 Have Fun with Math, Louis Grant Brandes, Publisher J. Weston Walch, Portland ME, 1988.
- Manipulatives: In this case, 36 blocks of different colors and shapes.

Description of Instruction:

1. The instructor led a discussion of the concept of fraction and naming fraction. She began by drawing on the board a large square divided into 16 smaller squares. First instructor asked how many squares there were. After everyone had time to make their own determination, the instructor asked for answers. The class then went through the process of finding all 30 squares. The group discussed the definitions and assumptions necessary to find all the squares, as well as the assumptions that limit our ability to find all the squares.

Participants used the same square to name various fractions and talk about whole and parts. The instructor asked questions about the square to help participants practice naming fractions, as: What is the whole thing here? How many parts is it divided into? One small square would be what fractional part of the whole? What if the square had been divided into only 4 equal parts? What if it had been divided into 4 unequal parts? What if each of the small squares were divided into two equal parts?

2. Instructor gave participants a lot of worksheets to help them with fractional concepts. The worksheets "*A Whole Thing ①*", "*Sequence A - Lesson 4*" (p. 17), and "*Jumping Floozie*" gave practice in naming fractions in situations where the whole or unit was known and already divided into equal parts. "*Jumping Floozie*" extended this to asking participants to locate the unit (whole one) when a particular part of it was given. "*Sequence A - Lesson 6*" (p. 23), as revised, gave participants an opportunity to draw 3 different types of illustrations for several tape measure fractions. Participants worked on "*Sequence A - Lesson 2*" (pp. 10-12) in small groups to identify whether the given drawings represented fractions (that is, a certain number of parts from a whole divided into equal parts). Also, "*A Whole Thing ②*" and "*A Whole Thing ③*" had participants name fractions based on units (wholes) of different sizes.
3. To demonstrate equivalent fractions, participants did a manipulative using 36 blocks of different colors and shapes. Participants gave a fraction name for various sets of blocks, for example, "What fraction of the blocks are large circles?" "What fraction of the blocks are purple?" Sometimes different people would offer different fractional names for the same group of blocks. The instructor and the class then discussed how each person arrived at their answer. Later, the instructor asked participants to try to group the blocks in more than one way and thus find different fractional names for that group.

For example, out of 36 blocks, 12 were large circles (that is $12/36$ of the blocks). The class figured out different ways to group and rename, such as $6/18$, $3/9$, $2/6$ and finally $1/3$ - the simplest way of naming that fractional part. The instructor used an accompanying worksheet called "*Equivalent Fractions*" to help focus the activity.

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4. Participants received copies of "Number." The instructor led discussion about it with such questions as "What is the whole here?" and "Which fractions are equivalent?". Participants then kept the handout as a reference.
5. Worksheet "Sequence C - Lesson 2" (p. 75) was used to give more practice with equivalent fractions.
6. When learning to compare fractions, the instructor led a discussion helping participants to use common sense to make those comparisons. For instance, when denominators are the same, a fraction with a larger numerator is larger. Or, in comparing $\frac{2}{5}$ and $\frac{5}{9}$, a participant might notice the $\frac{2}{5} < \frac{1}{2}$ while $\frac{5}{9} > \frac{1}{2}$. This approach was supported with worksheet "Sequence D - Lesson 6: Why do you think so?" (p. 99). Another useful worksheet for comparing fractions was "Sequence G - Lesson 1" (p. 141) which uses drawings to help participants visualize and compare fractions.
7. When participants seemed to have some understanding of the meaning of fractions and the relationships between fractions, the instructor introduced the mathematical concept of changing fractions to equivalent fractions with a common denominator as a way of comparing the fractions. One worksheet used for practice here was "Sequence G - Lesson 2" (p. 145). For additional practice with equivalent fractions as well as with mixed numbers/improper fraction conversions, participants worked on pages 4-6 of Fractions.
8. The instructor used "Thinking About Fractions" as a review of the fraction concepts and definitions covered so far in this section.
9. "Lineup with Fractions" was used as an extension of these activities for those participants who had mastered the other information on comparing fractions.

Comments:

- The objective of much of this section was to give participants an opportunity to see, in as many different contexts as possible, what fractions mean and how they work.
- Instructor spent at least two sessions on an overview and definition of fractions.
- Developing Mathematical Thinking has a number of other potentially useful worksheets for teaching and practicing with fraction concepts and computation.
- Some of the activities from Activity II: Working with Tape Measures and Measurements were interspersed with the above activities, as another (and very practical) way of seeing/using fractions.
- Many of the activities chosen used only (or mostly) tape measure fractions. This approach helped participants gain as much familiarity as possible with the fractions they needed to use on the job.

Performance Assessment Methods and Results:

- To assess participant performance, the instructor listened to see if the participant could explain the concepts we are working with to her or to other participants. She looked to

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see if the participants were responding appropriately on paper, and watched for the types of mistakes they made. Also, the review page "*Thinking About Fractions*" was helpful in assessing what participants were really understanding.

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Activity IV: Computations With Fractions

1. Adding and subtracting fractions
2. Multiplying and dividing fractions
3. Estimation and developing a sense of reasonable answers

Resources and Materials Needed:

- # "Fraction Tile" (page 54) from Aftermath IV, Seymour and others, Creative Publications, 1975.
- Manipulatives representing halves, thirds, fourths, and sixths for use with "Fraction Tile"
- Manipulatives: In this case, strips of paper pre-cut and marked with fractional names such as $\frac{3}{4}$, $\frac{5}{8}$, $\frac{3}{16}$ and etc..
- Number Power 2: Fractions, Decimals, and Percents, Jerry Howett, Contemporary Books, Inc., Chicago, IL 1988.
- # "The Large Square Represents One Unit" from Aftermath I, p. 81, Seymour and others, Creative Publications, 1975.
- # "Closer To" page 95, Problem Solving in Mathematics, Lane County Mathematics Project, Grade 7, Dale Seymour Publications, Palo Alto, CA, 1983.
- Manipulatives--small blocks are good--at least 25-30--to use for illustrating division.
- # "Computing with Fractions and Mixed Numbers" instructor-made guideline to help people organize their thinking when setting up fraction computations. Mary Mayfield, 1996.
- # "Giant Slalom" page 100, Aftermath IV, Creative Publications, Worth, IL.
- # "Smallest Answer" pages 91 and 93, Problem Solving in Mathematics, Lane County Mathematics Project, Grade 7, Dale Seymour Publications, Palo Alto, CA, 1983.

Description of Instruction:

1. Participants were encouraged to "play" with adding and subtracting fractions, using "Fraction Tile" worksheet. Participants completed this worksheet individually or in pairs using their understanding of fractions--before learning or reviewing any rules for adding and subtracting. They could also use the fraction manipulatives to visualize how the fractions would add together if they liked. Later the ideas gained by working without rules became part of the discussion when the instructor introduced the rules.
2. The instructor did some lecture and board work on common denominators. Then the group worked with the "Fraction Tile" worksheet using common denominators.

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3. Participants worked from appropriate pages in Number Power 2 to practice adding and subtracting fractions. Some participants also practiced with a few basic word problems.
4. Instructor brought in some strips of paper representing various tape measure fractions of a whole ($\frac{3}{4}$, $\frac{5}{8}$ etc.) that participants manipulated to practice adding fractions together.
5. The instructor introduced multiplying fractions first, and then worked with both multiplying and dividing fractions together. The worksheet "*The Large Square Represents One Unit*" was used to give a sense of the meaning of multiplication of fractions. The instructor introduced multiplication and division of fractions in two separate sessions, but the subjects were kept related. After the instructor led the class through working a few sample problems on the board, participants practiced on appropriate pages from Number Power 2.
6. Used the worksheet "*Closer To*" that gives examples of estimating with division involving mixed numbers. (For example, to divide 30 by $2\frac{2}{5}$, it gives a step-by-step procedure for estimating that answer.) People really have a hard time estimating with fractions, particularly multiplying and dividing with fractions. To help with estimating division involving fractions, the instructor sometimes used blocks to illustrate fractional concepts. Because the worksite works often with eighths, the instructor would (for example) designate one small block as $\frac{1}{8}$ and thus a group of 8 to represent 1 whole. Then, she (and later the participants) would lay out blocks to represent various fractions and mixed numbers. (For example, 3 groups of 8 blocks and three additional blocks to represent $3\frac{3}{8}$.) Then, she would illustrate (by grouping the blocks) various division problems, such as $3\frac{3}{8} \div 3$ or $3\frac{3}{8} \div \frac{1}{8}$ or $3\frac{3}{8} \div 1\frac{1}{8}$. This helped participants visualize what was happening with division of fractions. It also gave an opportunity to illustrate and discuss the two meanings of division (dividing a given quantity into a specified number of groups or dividing a given quantity into groups of a specified size). With whole numbers it was relatively easy to think of division in either way, but with fractions or mixed numbers, it was hard to illustrate or visualize dividing an amount into $\frac{1}{8}$ of a group or $1\frac{1}{8}$ groups. It was easier to think of the problem as dividing the amount into groups of a specified size (groups of $\frac{1}{8}$ each or $1\frac{1}{8}$ each). Although this certainly didn't create an "aha" for everyone, it did seem to help some people conceptualize and feel more comfortable with division.
7. For review and practice, participants used appropriate pages from Number Power 2 or other books, using the guidelines from "*Computing with Fractions and Mixed Numbers*" as necessary. When participants were feeling pretty competent, "*Giant Slalom*" provided a quick (and light) "test" of their skill. Another useful review was "*Smallest Answer*," which gave an opportunity to practice estimation and fraction sense as well as computation.

Comments:

- The instructor spent five sessions on computation with fractions.

Course Title: Fractions and Measurement in the Workplace

- Participants found the manipulative activity of adding strips of paper "hokey," and so it was allowed to die. "Don't you do these things with pencil and paper?" So, the class used standard practice pages instead. This was the instructor's first time to try to use this activity. It seems to be a good idea. Maybe with better preparation and presentation, it might have been better received.
- The purpose of "*The Large Square Represents One Unit*" activity was to help participants visualize multiplication of fractions. Probably everyone didn't come away with a mental image of multiplication of fractions, but it did seem to help those who "got it." In the future, instructor would supplement with a manipulative exercise.

Performance Assessment Methods and Results:

- "*Giant Slalom*" is a helpful and quick way to assess a participant's ability to compute with fractions.

Activity V: Word Problems Including Workplace Production Problems Using Fractions

1. Standard fraction word problems, chosen for their practicality and/or everyday usefulness
2. Word problems using fractions in workplace situations
3. Computations based on production sheets ("flyers," a type of workplace document, with dimensions and product requirements)

Resources and Materials Needed:

- # # "*Deciding When to Add, to Subtract, to Multiply or to Divide*," # # (This worksheet can be found in the Basic Math Curriculum) Mary Mayfield, 1996
- # "*Computing with Fractions and Mixed Numbers*" Mary Mayfield, 1996
- # "*Fraction Word Problem Practice*" Mary Mayfield, 1996
- # "*Workplace Practice - 1*" Mary Mayfield, 1996.
- # "*Workplace Practice - 2*" Mary Mayfield, 1996.
- # "*Workplace Practice - 3*" Mary Mayfield, 1996.

Description of Instruction:

1. Started with general word problems taken from the instructor's experience and math texts to familiarize participants with how word problems work. The problems were selected because they dealt with real situations that were generally familiar. Participants had an opportunity to practice with them, and get used to working with and making sense of word problems with fractions. "*Deciding When to Add, to Subtract, to Multiply or to Divide*" sheet was given out, discussed and used as a reference. It was used with "*Computing with Fractions and Mixed Numbers*" in solving the problems on "*Fraction Word Problem Practice*" worksheet.
2. In one session the participants were divided into groups of three or four. The groups worked to come up with at least two workplace word problems that would involve working with adding and subtracting fractions. Another session participants worked on coming up with workplace situations involving any kind of fraction computations. The instructor also asked each participant to bring their own workplace problems, so everyone could benefit from working with them. In 3-4 sessions participants worked in groups to solve the workplace problems that they and the instructor had devised. The problems came from different departments, and often required an explanation from one of the participants of what operations were involved and why. This was helpful in expanding the class' knowledge of company operations as well as honing their mathematical skills.

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3. After participants came up with the problems, the instructor worked on them between sessions. She had to make sure that all the information was there and would be clear to everyone. The resulting problems were used for practice in later classes. "*Workplace Practice - 1*" is an example of those problems. "*Workplace Practice - 2*" was created by one of the employees who served as a resource to the instructor. "*Workplace Practice - 3*" is an example of problems compiled by interviewing "leads" and other workers on the job.

Comments:

- Most of the rest of the sessions (6 to 8) were workplace-related word problems and computations.
- Participants had a very hard time coming up with fraction problems with a particular computational skill. Next time the instructor would only ask participants to come up with problem situations (without reference to how they would be computed), and would then categorize them as needed to use in instruction.
- The instructor encouraged participants to group themselves with people from different departments to give them an opportunity to teach each other. In a couple of cases participants refused to be parted from their friends, but generally the groupings were helpful.
- The instructor went into the workplace and learned basic information on setting up a rip-saw, a drill, and other equipment. From that knowledge, the instructor created problems related to the use of those machines.
- Additional work-related problems were devised by other company representatives who served as resources for the instructor. If the instructor had questions about material she was writing for class practice or about terminology or work procedures, etc., one of these representatives would help clarify the issues.

Course Title: Fractions and Measurement in the Workplace

Activity VI: Concluding Activities

1. Complete grant paperwork
2. Give post-test to participants
3. Provide feedback to participants, and get feedback from participants on class.

Materials and Resources Needed:

- Grant forms
- # "Fraction Posttest, Part 1" (computation), "Fraction Posttest, Part 2" (word problems, similar to those in the "Workplace Practice" series of worksheets)
- Individualized Learning Plans (with Part 1 already completed)

Description of Instruction:

1. Participants completed the paperwork for the grant.
2. Participants took the post-test.
3. Participants completed the Individualized Learning Plans.

Performance Assessment Methods and Results:

- Participants were given the opportunity to review the progress between the pre and post test results.
- Participants also had a chance to provide input to the instructor on the quality of the class.
- Part 1 of the Post-test was designed to "match" the pretest and to measure participant's gain in math skill since the pretest. Part 2 gave participants an opportunity to practice solving more complex work-related problems using the same set of mathematical skills.

Course Summary

Background:

Managers, lead supervisors, and shop-floor employees identified math as a major area needing training. At this secondary wood product company, employees need to use and read a tape measure to at least 1/16". Employees measure rough and finished dimensions of parts, identify drilling and boring locations, verify shaping dimensions etc. The Fractions/Measurement class was the first class offered at this site.

Expectations:

Company:

The company administers a math screening test to all temporary employees. The screening test includes many questions on fractions and measurement. However, despite the screening process, many temporary employees still have difficulty with measurement on-the-job. The class arose as a response to the need for further work on fraction and measurement skills.

Learner:

The class included both temporary and permanent employees. They self-selected for the class and attended after their work shift. Employees identified their need to read, measure, and compute fractions--skills needed to read the specifications of the work order, to identify parts, and to determine production requirements.

Instructor:

The instructor who has a strong background in teaching math skills at various levels, wanted to customize the class to the workplace. She consulted often with company personnel in order to tie the fraction and measurement concepts to the shop floor. She knew participants would learn about fractions, but wanted to include many workplace problems in order for participants to transfer that knowledge to their everyday work.

Course Title: Fractions and Measurement in the Workplace

Evaluations:

Company:

The Site Advisory Team decided to offer variations of the fraction/measurement class throughout the years of the project. The class has been offered to temporary employees in a similar format to the original class. It has also been combined with the flyer reading/document reading class and the English as a Second Language/flyer reading class.

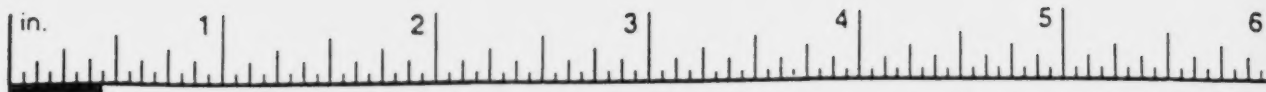
Learner:

Even though the class was offered after work shifts, attendance and completion rates were high. Participants responded that the class helped them understand math better, which in turn helped them to solve problems faster on-the-job.

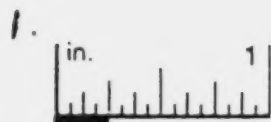
Instructor:

The first time through a customized class often only faintly approximates what happens after the instructor learns more about how the content relates to the workplace. This class fits this description. The instructor needed to learn enough about the the work done at the company to be able to customize the math skills required for the jobs. Helping participants understand the connections between what was done in class and what they did on the job proved challenging. By the time the class was taught again at this worksite, the instructor felt she had gained more information and was able to fit the many pieces of instruction together in a more cohesive way.

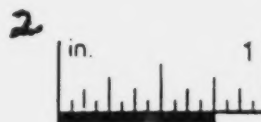
HANDOUTS



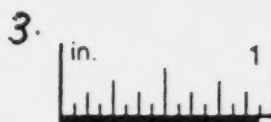
Example $\frac{7}{16}$



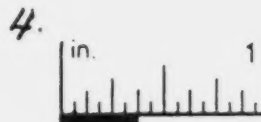
___ inch



___ inch



___ inch

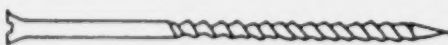


___ inch

Using an English system ruler, find the exact measurement of the washer (in diameter) and screw shown below.



5. ___



6. ___



What is the length of the tubing? ___



What is the length of the nail? ___

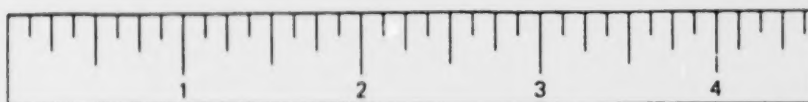


What is the length of the bolt

**Measurement
Practice - 1**

Name _____

1. A ruler whose unit of length is $\frac{1}{8}$ is shown. Use the ruler to find the lengths of these line segments:



- a.
- b.
- c.
- d.
- e.

2. A ruler whose unit of length is $\frac{1}{16}$ is shown. Use the ruler to find the lengths of these line segments:



- a.
- b.
- c.
- d.
- e.

3. Look at each line drawn below, and *estimate* its length. Fill in each estimate in the first column of the table provided.

- a.
- b.
- c.
- d.
- e.

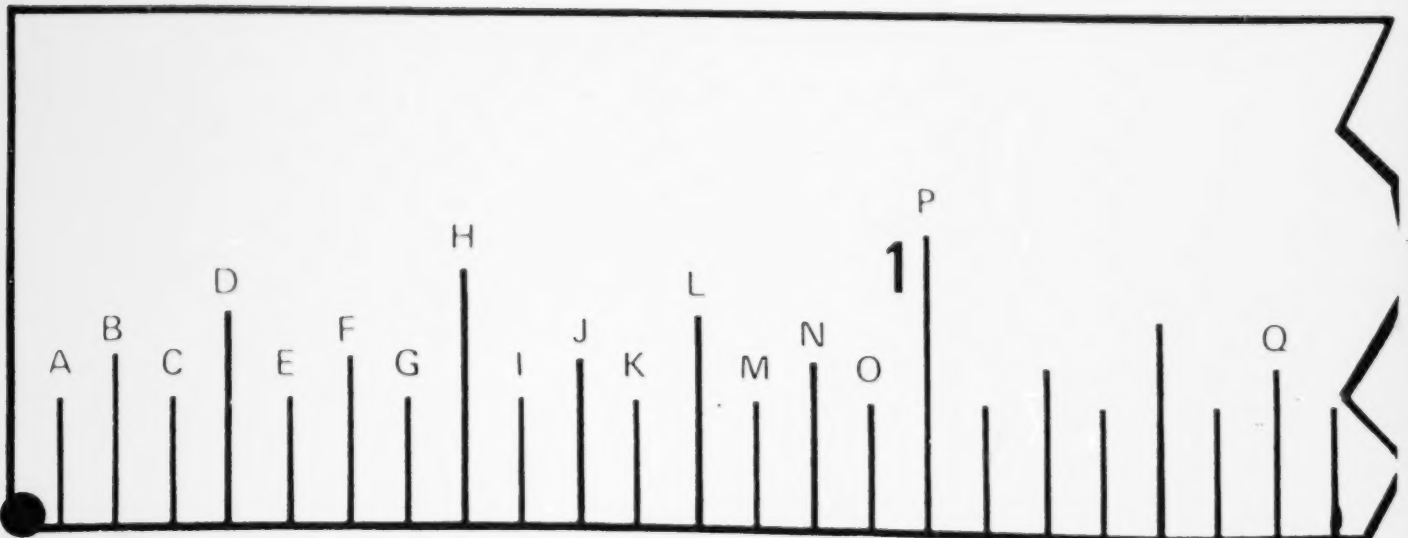
	Estimate	Measured Length
a.		
b.		
c.		
d.		
e.		

**Measurement
Practice - 2**

Eighths Rule

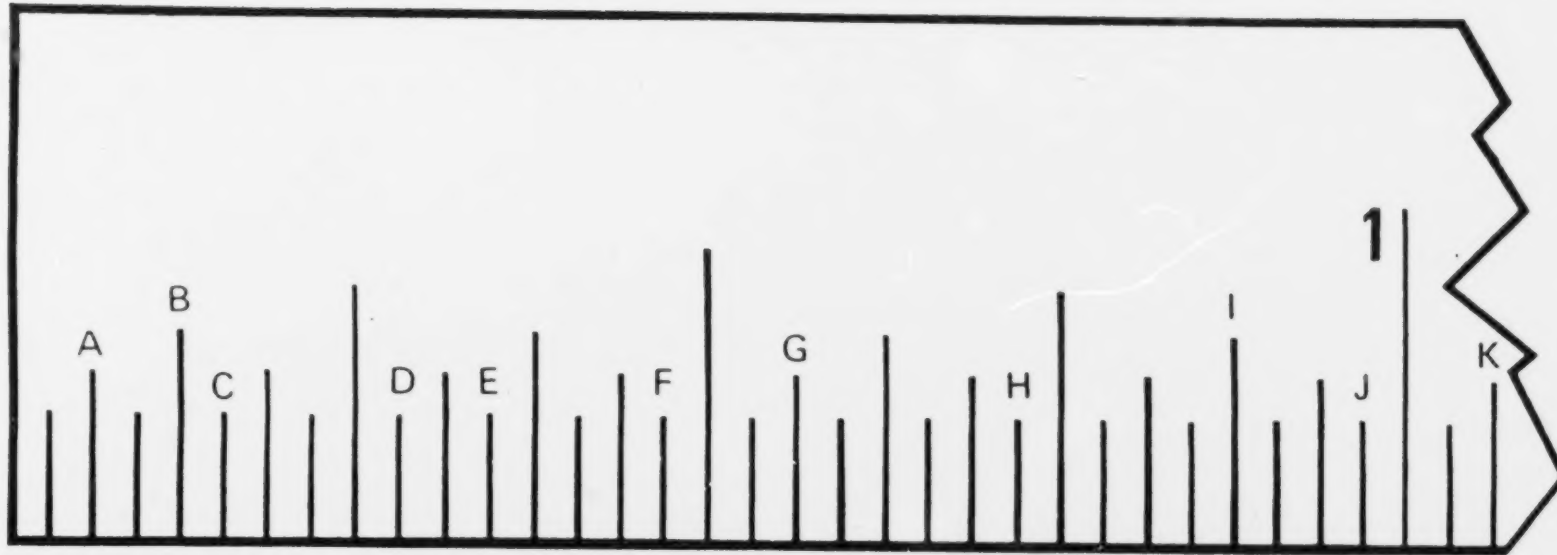


Sixteenths Rule



Thirty-Seconds Rule

Eugene, Oregon
Page B - 23



134

“GIANT INCHES” pg.2

135

Workplace Training Project
Lane Community College

Developing Fraction Sense --Ruler Exercise
Measure these line lengths

a) to the nearest $\frac{1}{2}$

inch

b) to the nearest $\frac{1}{4}$ inch

c) to the nearest $\frac{1}{8}$ inch

d) to the nearest $\frac{1}{16}$ inch

2. Find the sum of these fractions using the ruler.
Draw line segments and find their sum by measuring.

a) $\frac{3}{4} + \frac{1}{2}$

drawing:

b) $\frac{7}{8} + \frac{1}{2}$

drawing:

c) $2\frac{1}{2} + \frac{3}{4}$

drawing:

d) $1\frac{3}{4} + \frac{7}{8}$

drawing:

3. Work these mentally. Don't compute by hand or by calculator.

a) $2\frac{1}{4} + \frac{3}{4}$

b) $\frac{9}{16} - \frac{1}{2}$

c) $7\frac{3}{4} - \frac{1}{2}$

d) $4\frac{1}{6} + \frac{5}{6}$

4. Find these results using the ruler. Draw line segments and measure.

a) $\frac{1}{2}$ of $1\frac{1}{4}$

drawing:

b) $\frac{1}{4}$ of $4\frac{1}{2}$

drawing:

c) $2\frac{1}{2}$ of $1\frac{3}{8}$

drawing:

5. Work these mentally. Don't compute by hand or by calculator.

d) $\frac{1}{2}$ of $\frac{1}{4}$

e) $\frac{1}{2}$ of $2\frac{1}{2}$

e) $\frac{1}{2}$ of $\frac{2}{3}$

f) $\frac{1}{4}$ of $2\frac{1}{2}$

Tape Measure Math

Use a tape to determine which in each pair is larger.

1. $\frac{1}{4}$ or $\frac{1}{8}$ 2. $\frac{5}{8}$ or $\frac{5}{32}$ 3. $\frac{5}{8}$ or $\frac{7}{16}$ 4. $\frac{15}{16}$ or $\frac{7}{8}$

Use a tape to determine the following.

5. $\frac{24}{32}$ is equal to how many fourths? ____ 6. $\frac{10}{16}$ is equal to how many eighths? ____

7. $\frac{1}{4}$ is equal to how many 32nds? ____ 8. $\frac{7}{8}$ is equal to how many sixteenths? ____

9. $\frac{5}{16} + \frac{11}{16} =$ ____ 10. $\frac{3}{8} + \frac{3}{8} =$ ____ - 11. $\frac{3}{8} + \frac{7}{8} =$ ____

12. $\frac{3}{16} + \frac{9}{16} + \frac{8}{16}$ 13. $\frac{1}{4} + \frac{3}{8} =$ ____ 14. $\frac{3}{32} + \frac{15}{16} =$ ____

15. What is half of $\frac{1}{4}$? ____ 16. What is half of $\frac{3}{16}$? ____ 17. What is half of $\frac{7}{8}$? ____

18. What is half of $1\frac{1}{2}$? ____ 19. What is half of $2\frac{1}{4}$? ____

20. What is half of $2\frac{7}{8}$? ____

21. How many $\frac{1}{16}$ " in 2"? in 3"? in 10"? in $\frac{3}{4}$ "? in $1\frac{1}{4}$ "?

22. How many inches in $1\frac{1}{2}$ '? in $2\frac{1}{4}$ '? in $3\frac{3}{4}$ '?

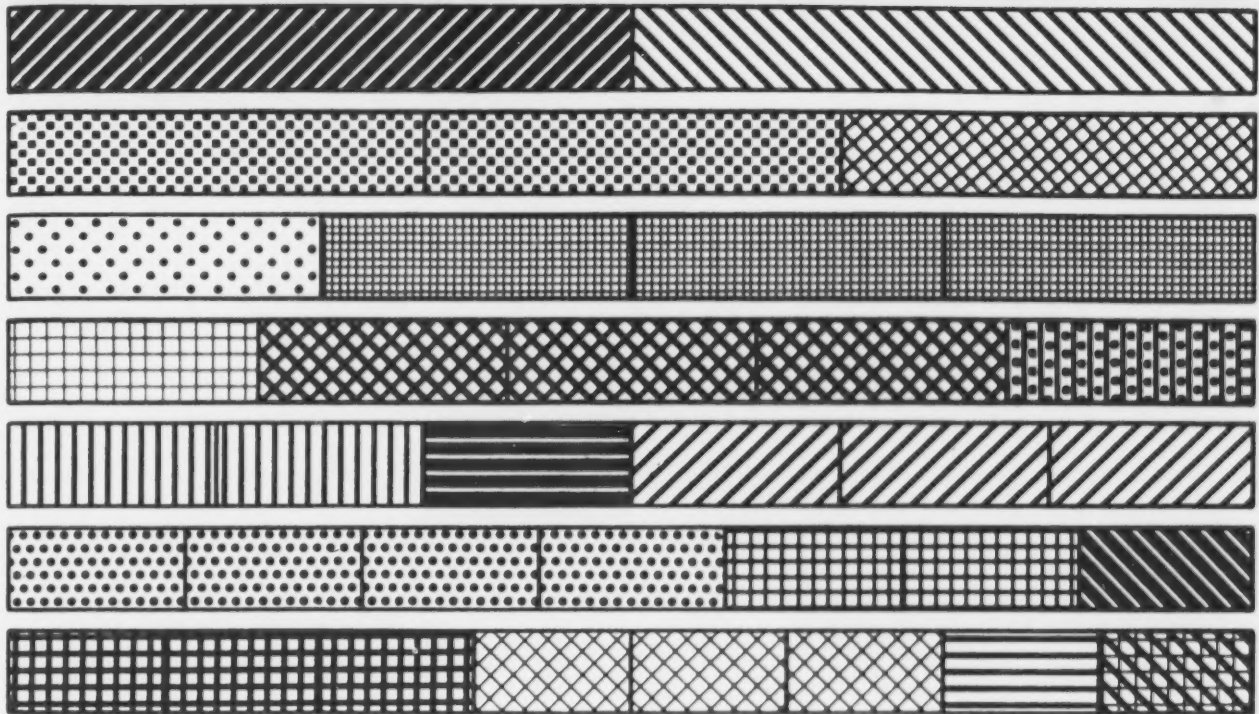
23. How many feet in 24"? in 30"? in 63"? in 126"?

A

WHOLE

THING

1



COMPLETE:

A  → 1E  → 2/3I  → O  → B  → 1/2F  → J  → P  → C  → G  → K  → Q  → D  → H  → L  → R  → M  → S  → N  → T  →

FRACTIONS UNIT

Sequence A - Lesson 4

NAME _____

In each problem, you are to illustrate and label each fraction below using the pictures at the bottom of the page. You may only use a fraction once, so you will need to choose your "1" carefully and then you will need to shade, or somehow emphasize the proper number of equal pieces so that your fraction is clearly pictured.

$$\frac{3}{4}$$

$$\frac{5}{12}$$

$$\frac{5}{6}$$

$$\frac{1}{2}$$

$$\frac{2}{3}$$

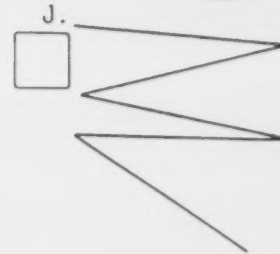
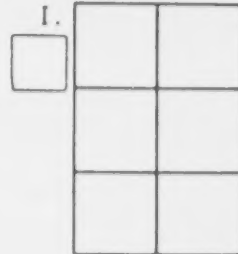
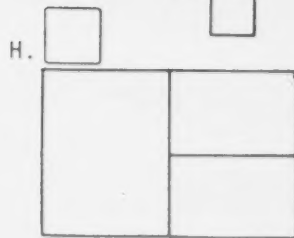
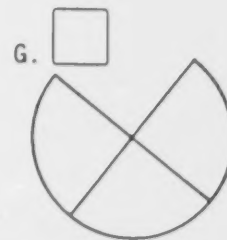
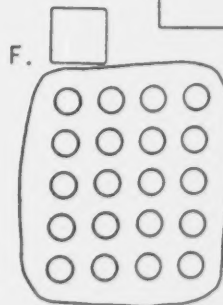
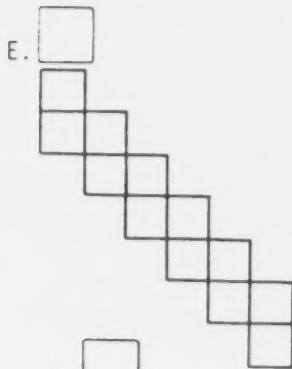
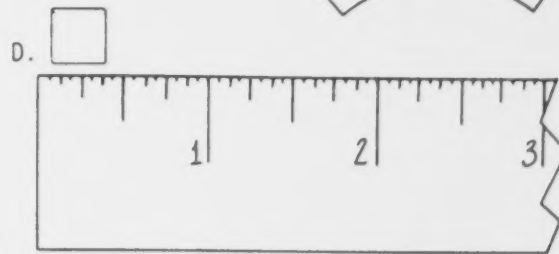
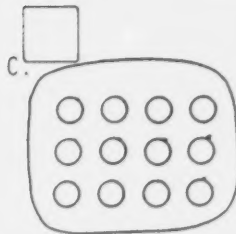
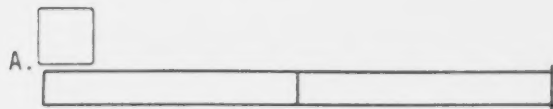
$$\frac{2}{4}$$

$$\frac{3}{8}$$

$$\frac{3}{10}$$

$$\frac{2}{5}$$

$$\frac{15}{16}$$

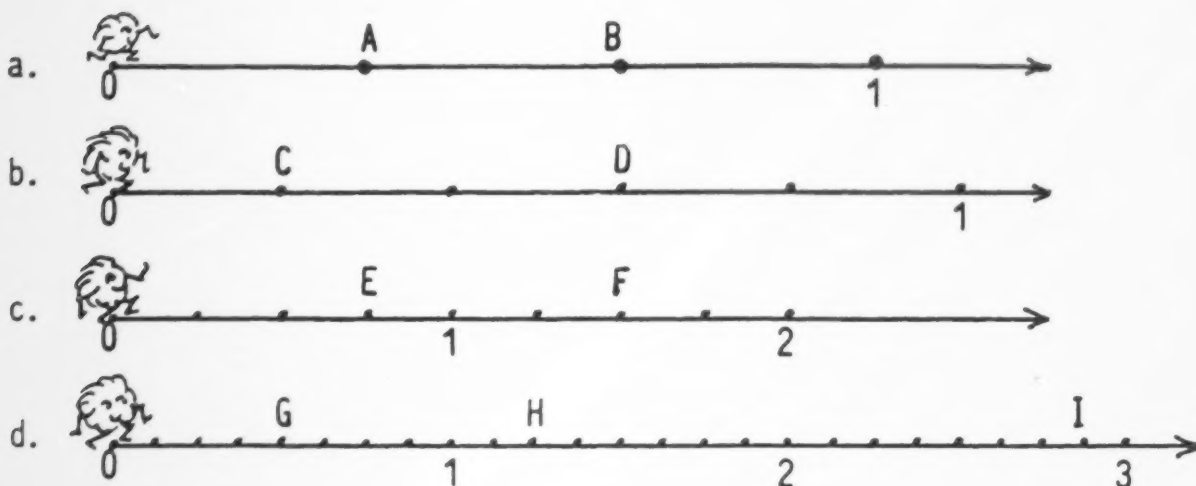


JUMPING FLOOZIE

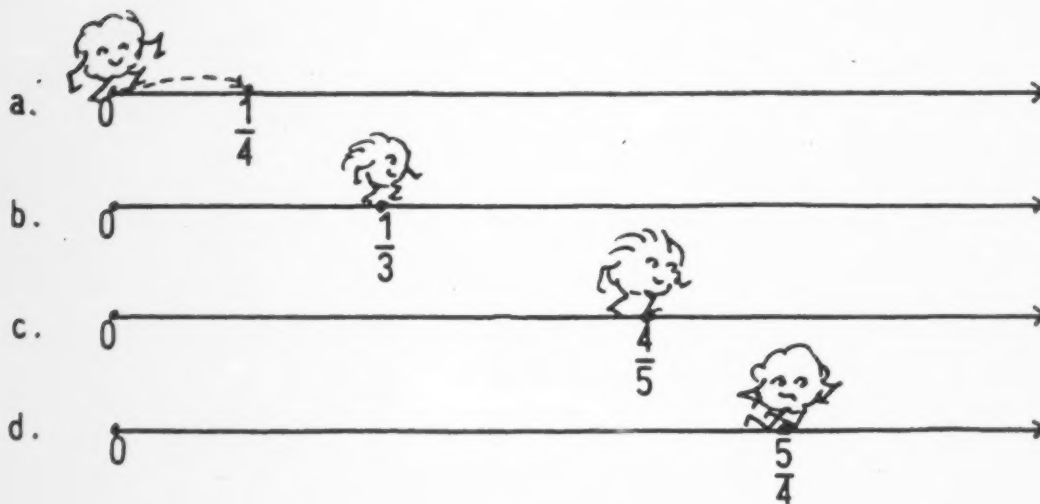
Floozie loves to jump. Sometimes he makes little jumps, sometimes big jumps, sometimes just medium-sized jumps. Most of the time he jumps forward, but sometimes he likes to jump backwards. As long as Floozie is jumping, he's happy.



1. The dots on the lines below show where Floozie landed. Write the fraction for each letter.



2. On the lines below, Floozie starts off with a jump. On his next jump he wants to land on the dot where $\frac{1}{4}$ should be. In each case show where $\frac{1}{4}$ should be located.



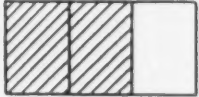
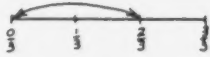
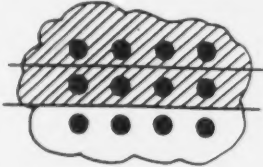
FRACTIONS UNIT

Sequence A - Lesson 6

What are Fractions?

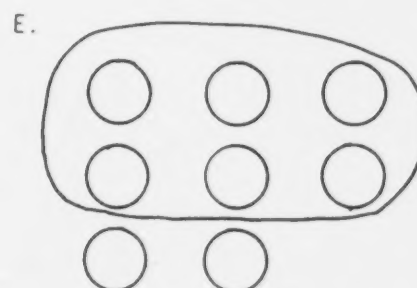
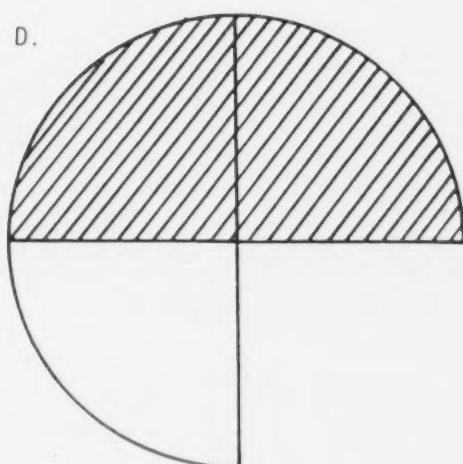
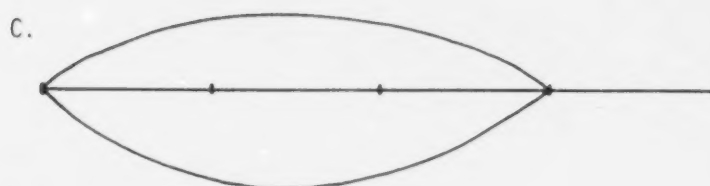
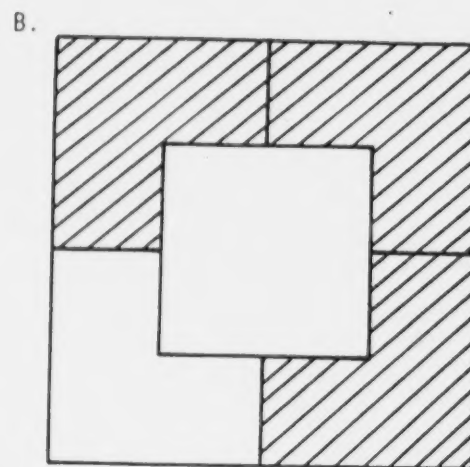
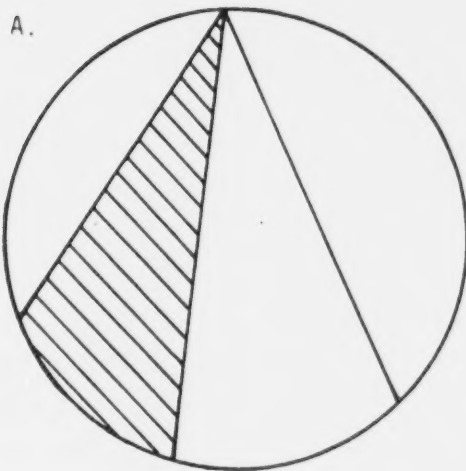
NAME _____

Draw a picture of each fraction below, using the 3 different fraction models:

Example: $\frac{2}{3} =$ 		
$\frac{1}{4} =$		
$\frac{3}{4} =$		
$\frac{5}{8} =$		
$\frac{3}{8} =$		
$\frac{1}{16} =$		
$\frac{5}{16} =$		

FRACTION UNIT

Sequence A - Lesson 2



FRACTIONS UNIT
Sequence A - Lesson 2

STUDENT WORKSHEET

NAME _____

a. Is it a fraction?

YES

NO

1. What's one? Draw a picture of it.
2. How many equal pieces has the one been broken into? ____ Draw a picture of one of the pieces.
3. How many of the equal pieces are being considered? ____
4. What's the fraction? 

1. There isn't a one that I can see ____
2. The one hasn't been broken into equal pieces ____
3. None of the equal pieces are emphasized ____

b. Is it a fraction?

YES

NO

1. What's one? Draw a picture of it.
2. How many equal pieces has the one been broken into? ____ Draw a picture of one of the pieces.
3. How many of the equal pieces are being considered? ____
4. What's the fraction? 

1. There isn't a one that I can see ____
2. The one hasn't been broken into equal pieces ____
3. None of the equal pieces are emphasized ____

FRACTIONS UNIT

Sequence A - Lesson 2

c. Is it a fraction?

YES

1. What's one? Draw a picture of it.
2. How many equal pieces has the one been broken into? ____ Draw a picture of one of the pieces.
3. How many of the equal pieces are being considered? ____
4. What's the fraction? 

NO

1. There isn't a one that I can see ____
2. The one hasn't been broken into equal pieces ____
3. None of the equal pieces are emphasized ____

d. Is it a fraction?

YES

1. What's one? Draw a picture of it.
2. How many equal pieces has the one been broken into? ____ Draw a picture of one of the pieces.
3. How many of the equal pieces are being considered? ____
4. What's the fraction? 

NO

1. There isn't a one that I can see ____
2. The one hasn't been broken into equal pieces ____
3. None of the equal pieces are emphasized ____

e. Is it a fraction?

YES

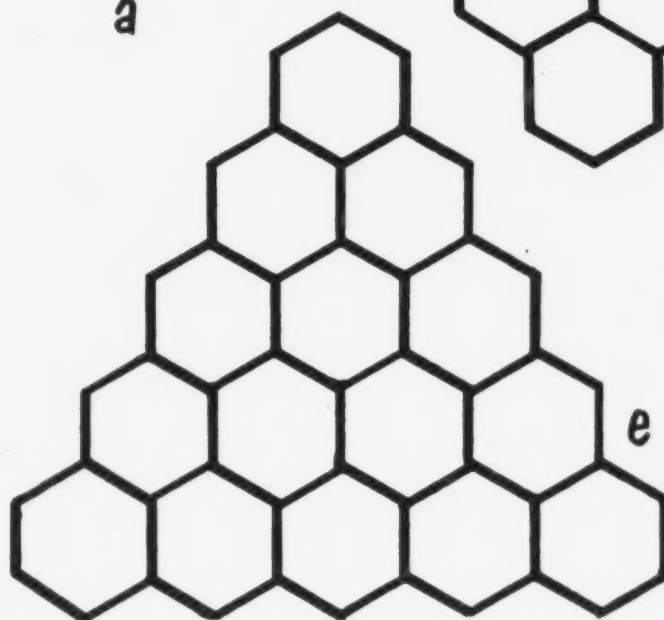
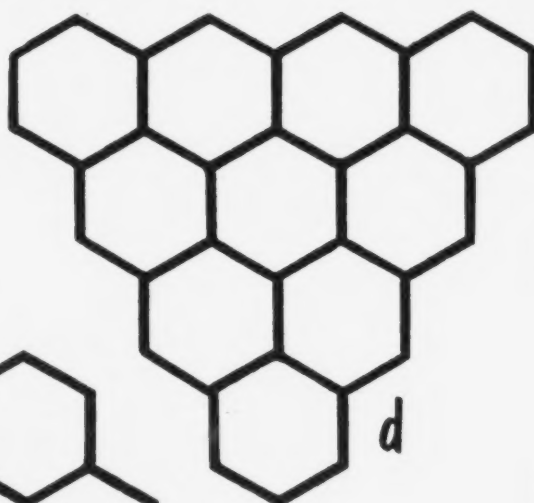
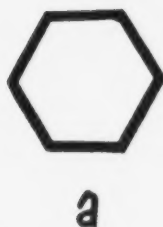
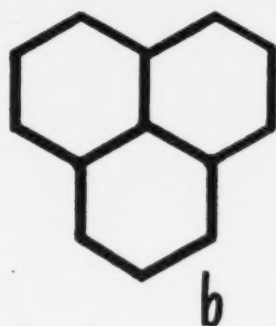
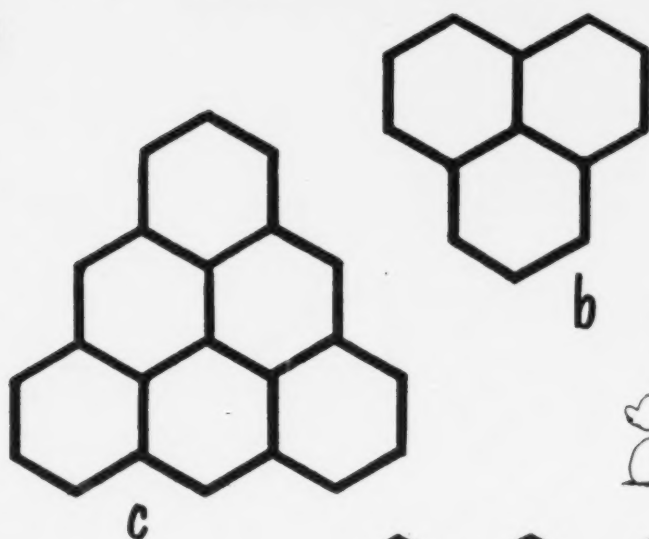
1. What's one? Draw a picture of it.
2. How many equal pieces has the one been broken into? ____ Draw a picture of one of the pieces.
3. How many of the equal pieces are being considered? ____
4. What's the fraction? 

NO

1. There isn't a one that I can see ____
2. The one hasn't been broken into equal pieces ____
3. None of the equal pieces are emphasized ____

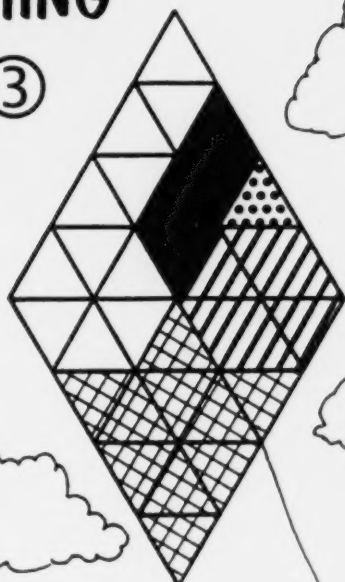
A WHOLE THING ②

IN EACH OF THE FIVE PROBLEMS BELOW, ONE OF THE FIGURES REPRESENTS ONE WHOLE THING. FILL IN THE NUMBERS REPRESENTED BY THE OTHER SHAPES.



A WHOLE THING

③



1. IF THE SHADED SECTION OF THE KITE REPRESENTS **ONE**, THEN:

1a. DOTTED REGION

1b. CROSSED REGION

1c. STRIPED REGION

1d. WHITE REGION

2. IF THE SHADED AREA IS 2:

2a. DOTTED REGION

2b. CROSSED REGION

2c. STRIPED REGION



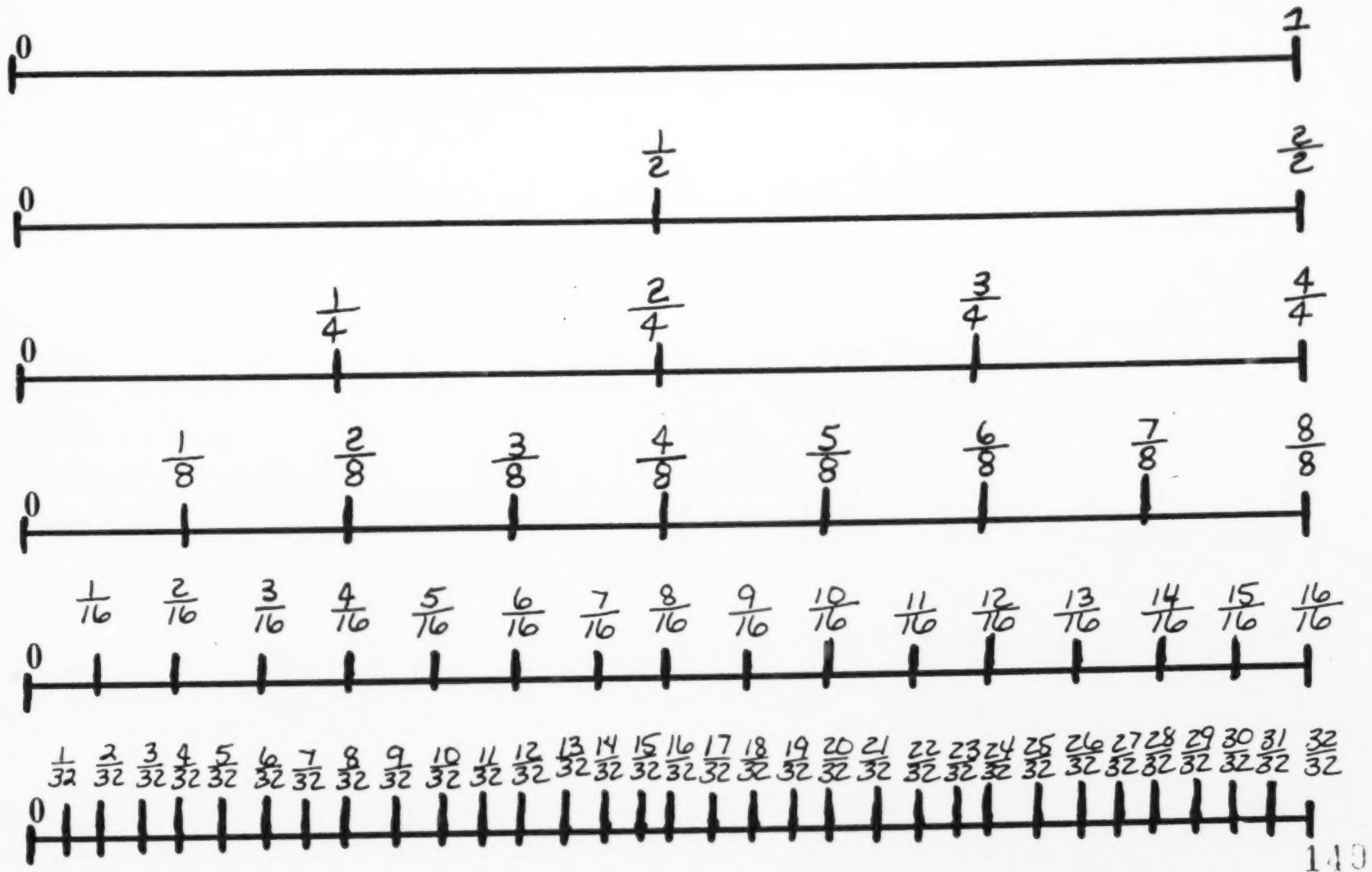
EQUIVALENT FRACTIONS

36

32

Large Circles	Rectangles
Purple	Large Circles
Small Circles	Purple
Yellow	Small Squares
Orange	Blue
Rhombi Parallelograms	Orange
Rounded	Right Angles
Triangles	Triangles
Blue / Yellow	Rounded
Equal Sides	Yellow / Green
At least one Right Angle	Red / Yellow
	Equal Sides
	At least one Right Angle

"NUMBER LINE"



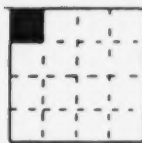
FRACTION UNIT

Sequence C - Lesson 2

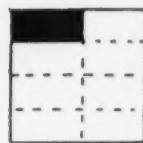
Name _____



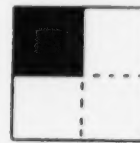
1



$\frac{1}{16}$



$\frac{1}{8}$



$\frac{1}{4}$



$\frac{1}{2}$

Use the pictures above to find the missing numbers in the following problems.

Remember, the problem : $\frac{1}{2} = \frac{\square}{4}$ can be read, "One-half is equivalent to how many fourths?"

a.) $\frac{1}{2} = \frac{\square}{4}$

b.) $\frac{1}{8} = \frac{\square}{16}$

c.) $\frac{\square}{8} = \frac{1}{4}$

d.) $\frac{1}{4} = \frac{\square}{16}$

e.) $\frac{\square}{8} = \frac{1}{2}$

f.) $\frac{\square}{4} = 1$

g.) $\frac{3}{8} = \frac{\square}{16}$

h.) $\frac{3}{4} = \frac{\square}{8}$

i.) $\frac{6}{8} = \frac{\square}{16}$

j.) $\frac{\square}{16} = \frac{2}{8}$

k.) $\frac{4}{4} = \frac{\square}{16}$

l.) $\frac{7}{8} = \frac{\square}{8}$

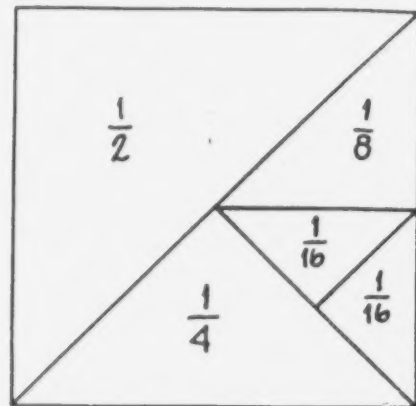
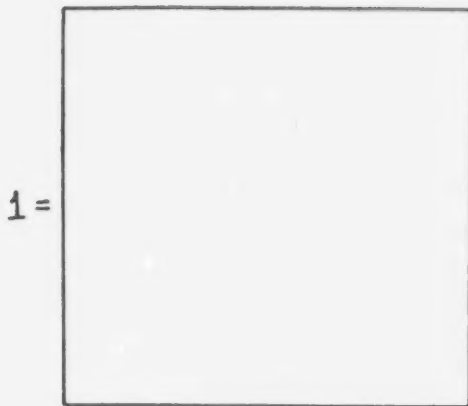
m.) $\frac{2}{2} = \frac{\square}{8}$

n.) $\frac{5}{8} = \frac{10}{\square}$

o.) $\frac{3}{4} = \frac{12}{\square}$

Sequence D - Lesson 2

Name _____



Cut these
pieces out

Use the 5 pieces cut-out above to help you place the proper symbol, $<$, $>$, or $=$ between the following fractions.

1) $\frac{2}{16}$ $\frac{1}{8}$	7) $\frac{3}{16}$ $\frac{2}{8}$	13) $\frac{5}{16}$ $\frac{2}{8}$
2) $\frac{2}{8}$ $\frac{6}{16}$	8) $\frac{3}{8}$ $\frac{6}{16}$	14) $\frac{4}{4}$ $\frac{9}{16}$
3) $\frac{10}{16}$ $\frac{5}{8}$	9) $\frac{5}{16}$ $\frac{1}{4}$	15) $\frac{13}{16}$ $\frac{3}{4}$
4) $\frac{2}{16}$ $\frac{2}{8}$	10) $\frac{3}{8}$ $\frac{5}{16}$	16) $\frac{3}{8}$ $\frac{2}{4}$
5) $\frac{1}{8}$ $\frac{4}{16}$	11) $\frac{7}{16}$ $\frac{2}{4}$	17) $\frac{2}{2}$ $\frac{5}{16}$
6) $\frac{1}{2}$ $\frac{3}{16}$	12) $\frac{9}{16}$ $\frac{4}{8}$	18) $\frac{7}{16}$ $\frac{3}{8}$

FRACTION UNIT

Sequence D - Lesson 6

Name _____

Use what you know about fractions to compare each pair of fractions below. Circle the one you think is LARGEST, and tell why you think it is Largest. Write " = " between the fractions you think are equal and tell why you think they're equal. You may draw pictures to help you if you need to. If you finish early, turn your paper over, and draw a picture of what you felt like while you were taking this quiz.

Why do you think so?

1. $\frac{1}{9}$ $\frac{1}{10}$

2. $\frac{3}{5}$ $\frac{14}{28}$

3. $\frac{7}{10}$ $\frac{11}{12}$

4. $\frac{8}{16}$ $\frac{9}{16}$

5. $\frac{4}{5}$ $\frac{5}{6}$

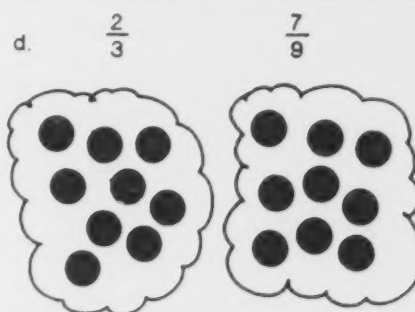
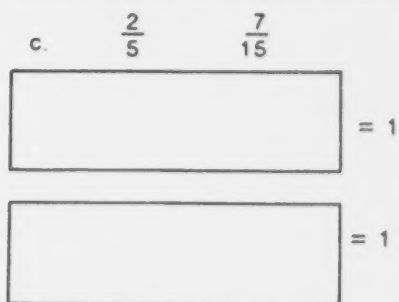
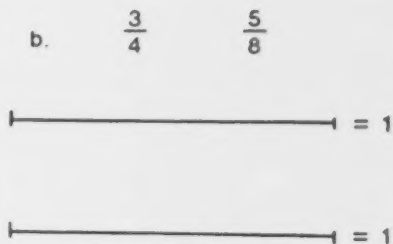
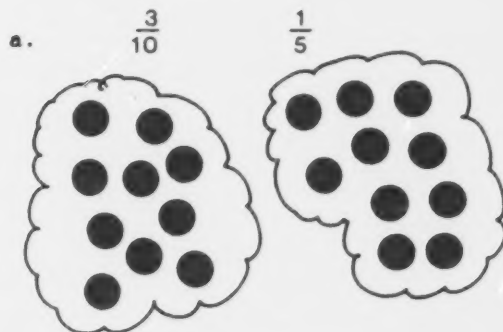
6. $\frac{7}{100}$ $\frac{7}{101}$

FRACTION UNIT

Sequence G - Lesson 1

Name _____

Use pictures to compare each pair of fractions. Be sure that you use the same "1" for each of the two fractions being compared.



Draw your own pictures for problems e, f, g, and h.



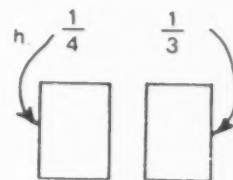
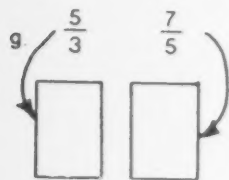
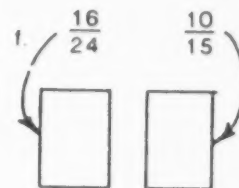
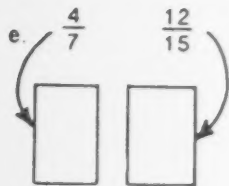
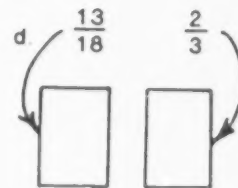
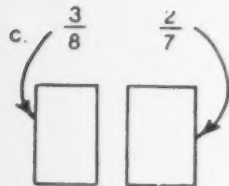
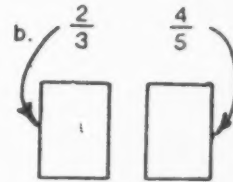
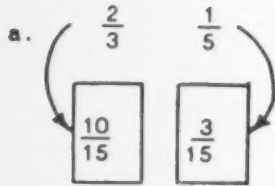
FRACTION UNIT

Sequence G - Lesson 2

Name _____

Change the following fractions into pairs of Like Fractions without changing their values. Then compare them by placing the proper symbol ($>$, $<$, $=$) between them.

Example:



"Thinking About Fractions"

What does a Fraction Mean??

The number on top is called the numerator.
The number on the bottom is called the denominator.

A fraction can be thought of as part of a whole.

The denominator tells how many equal parts the whole is divided into.
The numerator tells how many of those parts are shaded (or used).

A fraction can also be thought of as a division problem.

The line in a fraction means "divided by". (Notice that a fraction looks like a division sign with the dots replaced by numbers.)

The fraction $\frac{3}{4}$ can be read as "3 divided by 4". By dividing, you will find the decimal value of the fraction.

Problem 1: Use division to find the decimal value of the following fractions.

$\frac{1}{4}$	$\frac{4}{2}$	$\frac{12}{5}$
$\frac{1}{5}$	$\frac{4}{16}$	$\frac{12}{8}$
$\frac{1}{8}$	$\frac{4}{20}$	$\frac{12}{15}$

Notice the fractions whose value is greater than one.
Their numerators are larger than their denominators.

Finish this statement about the fractions that are less than one.

A fraction whose value is less than one has numerator that is _____ than its denominator.

Problem 3: Which of the fractions below are less than 1 in value? Which fractions are greater than 1? What is the value of the remaining fractions?

$\frac{2}{3}$	$\frac{5}{2}$	$\frac{4}{4}$
$\frac{6}{5}$	$\frac{3}{7}$	$\frac{10}{11}$
$\frac{7}{6}$	$\frac{7}{8}$	$\frac{11}{11}$

Problem 4: Insert numbers into the following fractions so that they are very close to, but less than, 1.

$\frac{\quad}{16}$	$\frac{\quad}{4}$	$\frac{\quad}{20}$	$\frac{13}{\quad}$	$\frac{7}{\quad}$	$\frac{31}{\quad}$
--------------------	-------------------	--------------------	--------------------	-------------------	--------------------

Fractions Close to $\frac{1}{2}$

The fractions $\frac{4}{8}$, $\frac{3}{6}$, $\frac{8}{16}$ are all equal to $\frac{1}{2}$. Half of the bottom number equals the top number.

If the numerator of a fraction is greater than $\frac{1}{2}$ of the denominator, the value of the fraction is greater than $\frac{1}{2}$.

Problem 5: Each of the following fractions is equal to or close to $\frac{1}{2}$ in value. Which fractions are exactly $\frac{1}{2}$ in value? Which are greater than $\frac{1}{2}$?

$\frac{3}{5}$

$\frac{6}{11}$

$\frac{35}{70}$

$\frac{2}{4}$

$\frac{34}{64}$

$\frac{5}{10}$

$\frac{7}{16}$

$\frac{5}{9}$

Comparing Fractions

If two fractions have the same denominator, the one with the larger numerator has the larger value.

If two fractions have the same numerator, the one with the smaller denominator has the larger value.

Sometimes you can reason in other ways to help you decide. For example, if you were to compare $\frac{8}{15}$ and $\frac{9}{19}$, you would notice that neither the numerators nor the denominators are the same. But the values of both fractions are close to $\frac{1}{2}$. $\frac{8}{15}$ is greater than $\frac{1}{2}$, and $\frac{9}{19}$ is less. So $\frac{8}{15}$ is larger.

Problem 6: Decide which fraction in each pair is larger. Try to do all of these mentally. Be prepared to explain your choice.

$\frac{4}{5}$ or $\frac{4}{7}$

$\frac{9}{10}$ or $\frac{10}{9}$

$\frac{8}{11}$ or $\frac{7}{11}$

$\frac{13}{5}$ or $\frac{13}{10}$

$\frac{5}{6}$ or $\frac{5}{8}$

$\frac{3}{16}$ or $\frac{6}{16}$

$\frac{5}{8}$ or $\frac{4}{9}$

$\frac{5}{9}$ or $\frac{8}{17}$

$\frac{3}{4}$ or $\frac{4}{3}$

$\frac{9}{32}$ or $\frac{9}{16}$

$\frac{7}{8}$ or $\frac{3}{8}$

$\frac{3}{8}$ or $\frac{9}{16}$

Lineup with Fractions

Draw a straight line through any three numbers in a block that are in order of their values (smaller to larger or larger to smaller). It may be possible to draw more than one straight line in a block. The straight lines may be horizontal, vertical, or diagonal. A lineup in the first block has been marked as an example. Try to find the lineups in the other blocks.

1

$\frac{1}{8}$	$\frac{1}{2}$	$\frac{1}{4}$
$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{2}$
$\frac{1}{12}$	$\frac{1}{4}$	$\frac{1}{8}$

2

$\frac{5}{8}$	$\frac{3}{8}$	$\frac{7}{8}$
$\frac{3}{4}$	$\frac{1}{8}$	$\frac{1}{4}$
$\frac{3}{8}$	$\frac{1}{2}$	$\frac{3}{4}$

3

$\frac{7}{10}$	$\frac{1}{2}$	$\frac{3}{10}$
$\frac{9}{10}$	$\frac{1}{5}$	$\frac{3}{5}$
$\frac{4}{5}$	$\frac{9}{10}$	$\frac{2}{5}$

4

$\frac{3}{4}$	$\frac{1}{4}$	$\frac{7}{12}$
$\frac{1}{12}$	$\frac{1}{2}$	$\frac{1}{3}$
$\frac{1}{6}$	$\frac{2}{3}$	$\frac{5}{12}$

5

$\frac{1}{4}$	$\frac{5}{8}$	$\frac{1}{2}$
$\frac{5}{12}$	$\frac{7}{12}$	$\frac{1}{6}$
$\frac{11}{12}$	$\frac{2}{3}$	$\frac{3}{4}$

6

$\frac{9}{16}$	$\frac{3}{8}$	$\frac{1}{2}$
$\frac{1}{8}$	$\frac{7}{16}$	$\frac{3}{4}$
$\frac{7}{8}$	$\frac{1}{4}$	$\frac{5}{8}$

7

$\frac{5}{8}$	$\frac{3}{16}$	$\frac{3}{8}$
$\frac{1}{2}$	$\frac{7}{8}$	$\frac{5}{16}$
$\frac{3}{4}$	$\frac{1}{8}$	$\frac{1}{4}$

8

$\frac{5}{8}$	$\frac{5}{24}$	$\frac{1}{4}$
$\frac{11}{12}$	$\frac{2}{3}$	$\frac{5}{6}$
$\frac{1}{8}$	$\frac{1}{12}$	$\frac{3}{4}$

9

$\frac{1}{3}$	$\frac{3}{8}$	$\frac{5}{12}$
$\frac{7}{12}$	$\frac{7}{8}$	$\frac{1}{2}$
$\frac{7}{24}$	$\frac{1}{12}$	$\frac{1}{6}$

10

$\frac{5}{24}$	$\frac{3}{8}$	$\frac{1}{6}$
$\frac{1}{2}$	$\frac{7}{12}$	$\frac{5}{12}$
$\frac{1}{10}$	$\frac{3}{4}$	$\frac{1}{5}$

11

$\frac{5}{6}$	$\frac{3}{10}$	$\frac{2}{3}$
$\frac{2}{5}$	$\frac{5}{8}$	$\frac{1}{4}$
$\frac{3}{5}$	$\frac{7}{24}$	$\frac{4}{5}$

12

$\frac{7}{24}$	$\frac{1}{5}$	$\frac{9}{10}$
$\frac{7}{10}$	$\frac{1}{2}$	$\frac{1}{4}$
$\frac{7}{12}$	$\frac{11}{12}$	$\frac{1}{3}$

FRACTION TILE



USE THREE OF THESE
FOUR TILES TO MAKE
A TRUE NUMBER
SENTENCE.



1. $\square - \square + \square = \frac{7}{12}$

2. $\square + \square - \square = \frac{5}{12}$

3. $\square + \square + \square = 1$

4. $\square - \square - \square = 0$

5. $\square + \square - \square = \frac{11}{12}$

6. $\square + \square - \square = \frac{1}{12}$

7. $\square + \square - \square = \frac{2}{3}$

8. $\square + \square + \square = \frac{11}{12}$

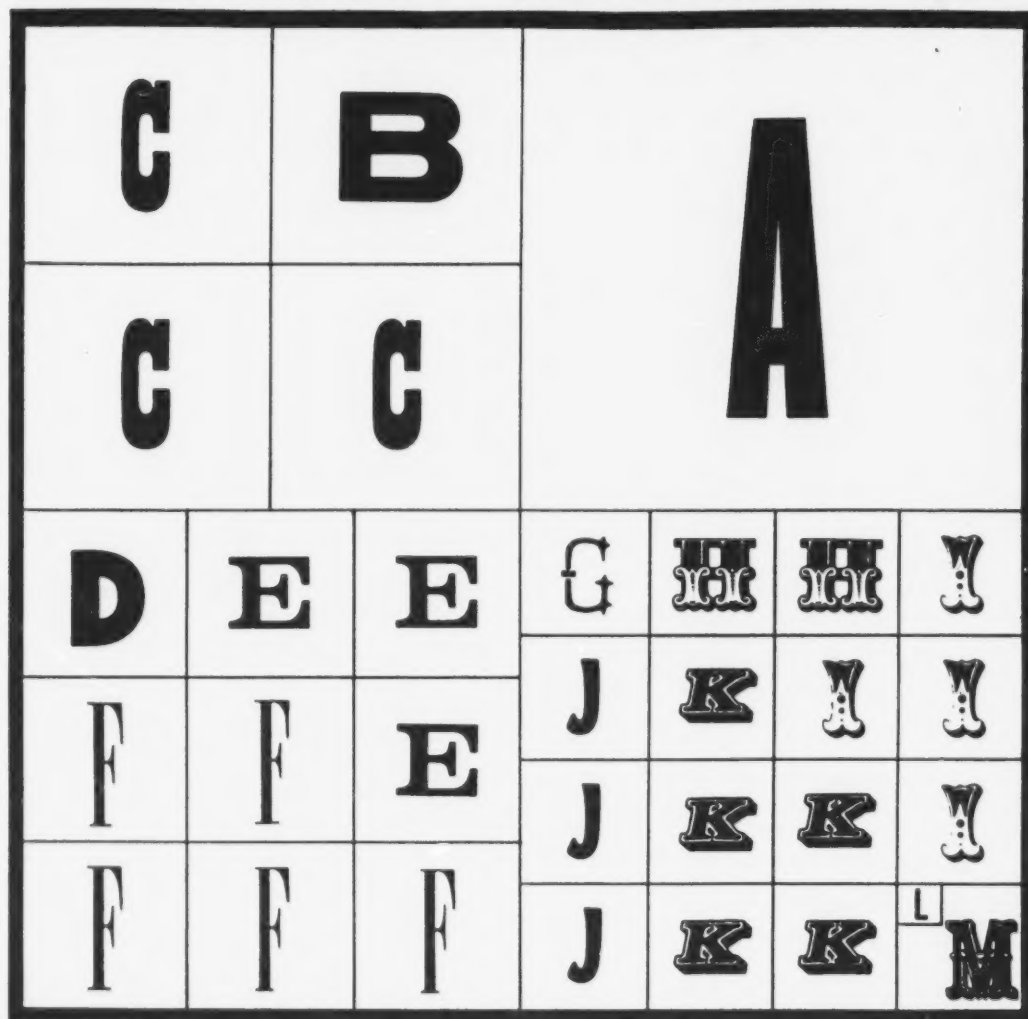
ONE OF THESE
CAN'T BE DONE.



PRETTY SNEAKY.



THE LARGE SQUARE REPRESENTS ONE UNIT



COMPLETE: D = ___ OF ___ = ___ H = ___ OF ___ = ___

A = $\frac{1}{4}$ OF 1 = $\frac{1}{4}$ E = ___ OF ___ = ___ I = ___ OF ___ = ___

B = $\frac{1}{4}$ OF $\frac{1}{4}$ = $\frac{1}{16}$ F = ___ OF ___ = ___ J = ___ OF ___ = ___

C = $\frac{3}{4}$ OF $\frac{1}{4}$ = ___ G = ___ OF ___ = ___ K = ___ OF ___ = ___

L = ___ OF ___ OF ___ = ___ M = ___ OF ___ OF ___ = ___

CLOSER TO

John and Sue were discussing their estimates for $12 \div 2\frac{5}{8}$. John estimated the answer to be 6. Sue estimated the answer to be 4. Who had the better estimate?

The teacher said that $12 \div 2\frac{5}{8} = 4\frac{4}{7}$. Since Sue's estimate was closest, she was asked to explain her procedure. Sue wrote this on the chalkboard.

$$12 \div 2\frac{5}{8}$$

$12 \div 2$	$12 \div 3$
6	4
4	

$2\frac{5}{8}$ is between (2) and (3)

so the answer is between 6 and 4

Since $2\frac{5}{8}$ is closer to 3, the answer is closer to 4.



My procedure also works for multiplication.

Study Sue's procedure. Use it to find a good estimate for each of these problems.

1. $30 \div 2\frac{2}{5}$

2. $20 \div 4\frac{4}{5}$

3. $24 \div 3\frac{1}{8}$

4. $9 \times 3\frac{5}{6}$

5. $5\frac{1}{3} \times 8$

6. $3\frac{3}{5} \times 6$

7. $4 \times 8\frac{7}{10}$

8. $12\frac{1}{12} \times 5$

9. $36 \div 3\frac{2}{5}$

COMPUTING WITH FRACTIONS AND MIXED NUMBERS

Add/Subtract

1. Write the numbers to be added or subtracted in a column (one above the other).
2. Find a common denominator and changed each fraction to an equivalent fraction with that denominator.
3. Add or subtract.
 - a) If adding mixed numbers and the fractional answer is an improper fraction, change to mixed number (by dividing), then add the whole number into the whole number column.
 - b) If subtracting mixed numbers and the top fraction is smaller than the bottom, borrow 1 whole from the whole number column, making sure to translate it into a unit fraction ($3/3$, $8/8$, etc.) that matches your common denominator. Then subtract.
4. Simplify. (Reduce.)

Multiply/Divide

1. Write the numbers to be multiplied or divided in a row (one after another).
2. Write each number as a fraction. (A whole number goes over 1; $5 = 5/1$. To change a mixed number to a fraction, multiply the whole number by the denominator, add the numerator, put that answer over your denominator; $5 - 3/4 = 23/4$.)
3. If dividing, change to multiplication and invert the second fraction.
4. Multiply.
5. Simplify. (Reduce.)

GIANT SLALOM

TRY THIS COMPUTATION
RACE TO CHECK
YOUR SPEED
AND
ACCURACY!

YOU ARE DISQUALIFIED
IF YOU DON'T FINISH
WITH THE CORRECT
ANSWER.

START
WITH
 $\frac{3}{8}$

1. MULTIPLY
BY
 $\frac{1}{2}$

2. ADD
 $\frac{1}{16}$

3. DIVIDE
BY
 $\frac{3}{4}$

4. SUBTRACT
 $\frac{1}{6}$

5. MULTIPLY
BY
 $\frac{3}{8}$

6. DIVIDE
BY
 $\frac{1}{8}$

7. SUBTRACT
 $\frac{1}{3}$

8. ADD
 $\frac{5}{12}$

FINISH

MY FINAL
ANSWER IS

SMALLEST ANSWER

Use only the numbers given.

Use each of them only once.

Place them in the boxes to get the smallest possible answer.

1. $\frac{\square}{\square} + \frac{\square}{\square} =$

Use
1, 2, 3, 4

2. $\frac{\square}{\square} - \frac{\square}{\square} =$

Use
1, 2, 3, 4

3. $\frac{\square}{\square} \times \frac{\square}{\square} =$

Use
1, 2, 3, 4

4. $\frac{\square}{\square} \div \frac{\square}{\square} =$

Use
1, 2, 3, 4

Smallest Answer (cont.)

5. $\frac{\square}{\square} - \frac{\square}{\square} =$

Use
1, 2, 4, 8

6. $\frac{\square}{\square} \times \frac{\square}{\square} =$

Use
1, 2, 4, 8

7. $\frac{\square}{\square} + \frac{\square}{\square} =$

Use
2, 3, 4, 12

8. $\frac{\square}{\square} - \frac{\square}{\square} =$

Use
2, 3, 4, 12

9. $\frac{\square}{\square} \times \frac{\square}{\square} =$

Use
2, 3, 4, 12

10. $\frac{\square}{\square} + \frac{\square}{\square} + \frac{\square}{\square} =$

Use
1, 2, 3, 4, 6, 12

Fraction Word Problem Practice

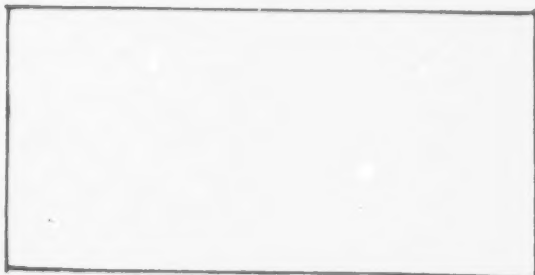
1. A table top is $27\frac{1}{2}$ inches from the floor. There is a shelf $12\frac{3}{4}$ inches above the table top. The shelf is how many inches from the floor?
2. Juanita works a $37\frac{1}{2}$ hour week. By Thursday, she had worked $34\frac{1}{4}$ hours. How many hours did she need to work on Friday?
3. The Swensons hiked $39\frac{3}{8}$ miles in three days. They hiked $12\frac{1}{4}$ miles on the first day and $14\frac{5}{8}$ miles on the second day. How many miles did they hike on the third day?
4. If you maintain a speed of 56 miles per hour for $3\frac{3}{4}$ hours, how far have you travelled in that time?
5. A stack of plywood measure 25 inches high. Each sheet of plywood is $\frac{5}{8}$ inch thick. How many sheets are in the stack?
6. At 9:00 a.m., the deli had a tube of salami that weighed $5\frac{5}{8}$ pounds. At closing, the remaining salami weighed $1\frac{3}{4}$ pounds. How much salami was sold that day?
7. The drapes made for Elaine's window will extend $1\frac{1}{2}$ feet on each side of the window frame. If the window frame measures $6\frac{1}{2}$ feet, how long should the drapery rod be?
8. The gasoline tank in Barry's car holds $15\frac{3}{4}$ gallons. When it is $\frac{1}{2}$ full, how many gallons does it contain?
9. A developer owns a 36-acre parcel, which he intends to partition into home lots. Each lot must be at least $\frac{3}{4}$ acre. How many homes can be built on this parcel?
10. A bolt whose shaft is $3\frac{1}{2}$ inches long is placed through a beam that is $2\frac{3}{8}$ inches thick. How far does the bolt extend on the other side of the beam?
11. The paving crew has $\frac{7}{10}$ mile of road to pave by Friday. The crew hopes to complete $\frac{2}{3}$ of this on Wednesday. What length of road should be paved by Wednesday evening?
12. Trish, a housepainter, painted $6\frac{1}{2}$ wall panels in one hour. At this rate, how many complete wall panels can she expect to paint in $7\frac{3}{4}$ hours?

13. Nora plans to use washers as spacers to raise the corner of an uneven table by $\frac{5}{8}$ inch. How many $\frac{1}{16}$ -inch-thick washers will she need to use to level the table?
14. A coil of rope 50 feet long is to be cut into pieces $4\frac{1}{2}$ feet long. How many pieces of this length can be cut?
15. A recipe for one loaf of bread calls for 1 cup of flour. A baker wants to make a dozen loaves. If he has only $5\frac{1}{3}$ cups of rice flour, how many more cups of this flour does he need?
16. A $\frac{2}{3}$ majority of those voting in the House of Representatives is needed to override a presidential veto. If all 435 representatives vote, how many votes are needed to override a veto?
17. To make curtains for their home, the Brechts bought a $17\frac{1}{2}$ yard bolt of fabric. If it takes $3\frac{1}{4}$ yards to make curtains for each window, how many windows do the Brechts have enough fabric for?
18. A carpenter needed pieces of molding $28\frac{1}{2}$ " long, $31\frac{1}{4}$ " long and $10\frac{1}{8}$ " long. If molding is only sold by the full foot, how many feet must she buy to complete this job?
19. Wallpaper comes in rolls that are $1\frac{3}{4}$ ' wide. How many side-by-side strips of wallpaper will Ray need to paper the wall of his den, which is 15' wide?
20. A survey found $\frac{3}{4}$ of employed young people own a VCR. Of these, $\frac{1}{4}$ also own a video camera. What fraction of employed young people also own a video camera?

"Workplace Practice - 1"

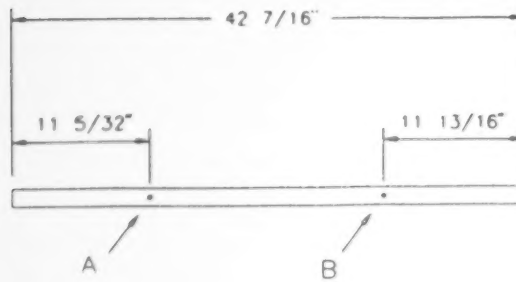
Try the following problems. Please make drawings if it will help you to visualize and understand the situation.

1. If you can only reduce by $\frac{1}{16}$ " at a time, how many passes through the Time-saver would it take to reduce a 2" x 2" piece of oak to $1\frac{5}{8}$ " x $1\frac{5}{8}$ "?
2. A WDB-4634-slat is $46\frac{3}{4}$ " long with a hole in the center. You need one $39\frac{1}{2}$ " long. How much must you trim off each end to keep the hole centered?
3. A Beer platform measures 21" x 52". The skirt for it is $18\frac{1}{4}$ " x $47\frac{3}{4}$ ". It needs to be centered on the platform. How much inset will there be from each side?
4. Doug was carrying a butcher block shelf measuring $4\frac{3}{4}$ " long when he tripped over a pallet and dropped the shelf. A piece $6\frac{9}{16}$ " broke off one end/edge. Rather than replace that piece, he trimmed $6\frac{9}{16}$ " off the length of the shelf. What was the length of the new shelf?
5. What is the maximum number of pieces $25\frac{1}{2}$ " x $45\frac{1}{4}$ " that can be cut from a 4' x 8' piece of $\frac{3}{4}$ " MDF? If you cut the maximum number of pieces from the 4" x 8" sheet, what will be the dimensions of the largest piece of waste board? (Allow $\frac{1}{8}$ " for saw kerf.) Use one of the rectangles below to draw the cuts that would need to be made.
6. What is the maximum number of pieces $15\frac{5}{8}$ " x $31\frac{3}{4}$ " that can be cut from a 4' x 8' piece of $\frac{3}{4}$ " MDF? Use one of the rectangles below to draw the cuts that would need to be made. If you were using oak plywood instead of MDF, what would be the maximum number of pieces you could cut from a 4' x 8' piece? What would be the dimensions of the largest piece of waste board? Again, use a rectangle below to draw the appropriate cuts. (And allow $\frac{1}{8}$ " for saw kerf.)



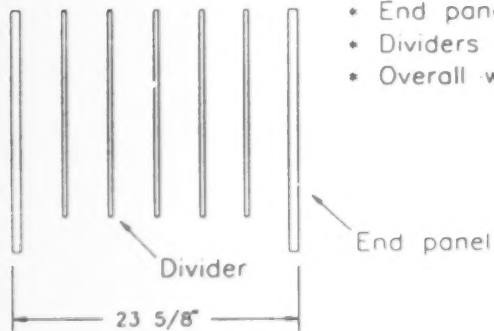
1. What is the Distance between
hole A and hole B? _____

"Workplace Practice - 2"



Ex: Wine Baskets

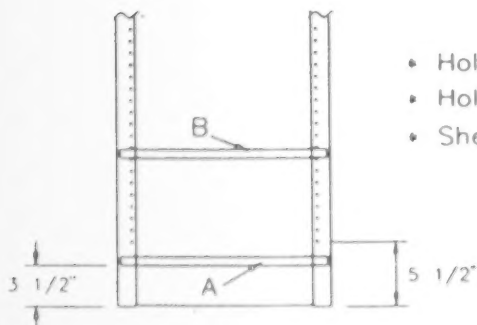
2. Divide the openings equally. What
is this dimension? _____



- End panel - $\frac{3}{4}$ " thick
- Dividers - $\frac{1}{2}$ " thick
- Overall width - $23 \frac{5}{8}$ "

Ex: Wine Island

3. What is the dimension from the top
of SHELF A, to the bottom of shelf
B? _____



- Holes are 1" O.C.
- Holes are $\frac{7}{32}$ " Diameter
- Shelf is $\frac{3}{4}$ " Thick

Ex: Shelving divider
openings

“Workplace Practice - 3”

You are setting up the rip saw. You need spaces of $2\frac{1}{4}"$, $1\frac{5}{16}"$, $1\frac{1}{2}"$ and $1\frac{5}{8}"$. You have spacers of $\frac{1}{16}"$, $\frac{1}{8}"$, $\frac{1}{4}"$, $\frac{1}{2}"$ and $1"$. Give at least 2 different combinations of spacers that will give you each necessary space.

You need to cut boards with the following measurements. First, however, arrange the measurements in descending order by width.

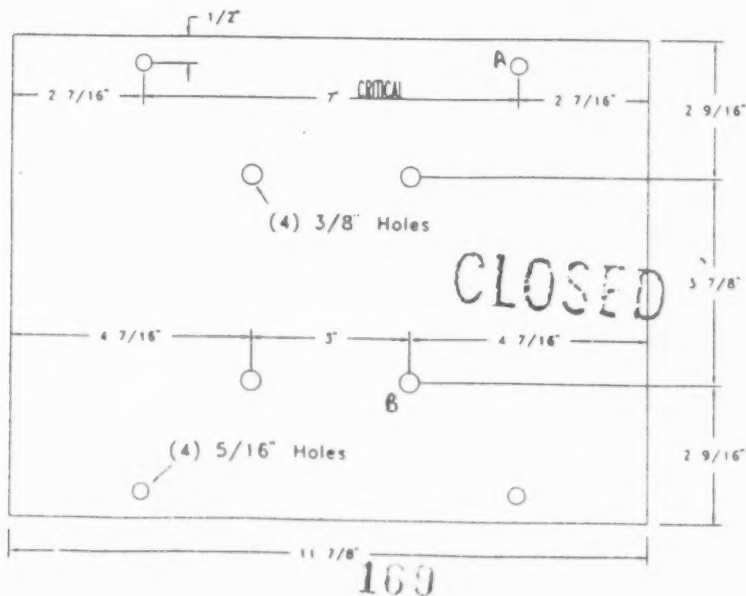
- 1-5-16" x 32"
- 1-1/2" x 72"
- 4" x 48"
- 2-1/4" x 47-1/2"
- 1-5/8" x 13"
- 1-5/16" x 23"

You are ripping a piece of 4'x8' plywood in half so that each piece will be 8' long. If the saw kerf (width of the saw blade) is $\frac{1}{8}"$, what is the actual width measurement of the 2 pieces you have cut?

Note: The final two problems will refer to the drawing at the bottom of the page.

On the flyer, specs are given as shown, for a board $11\frac{7}{8}"$ long and with measurements between holes as shown on the right. How wide will this board need to be?

It is discovered that this board is too long. Fortunately, the spacing between all the holes is correct. If $\frac{1}{4}"$ is trimmed off each end of the board, what will be the measurement from the end to hole A? from the end to hole B? How long will the resulting board be?



Fraction Posttest, Part 1

You can call this a test is you like, but please don't stress about it.

Please show your work on the separate sheet of lined paper you will be given.

1. Without measuring, how long do you think this line is? _____
2. Now, draw a line (again without measuring) approximately $5/8$ " long.
3. Which is larger, $3/8$ or $3/16$? _____ Explain. _____
4. Which is smaller, $3/4$ or $23/32$? How did you decide? _____
5. Reduce $40/64$. _____
6. Change $23/8$ to a mixed number. _____
7. $3/32 + 19/32 + 10/32 =$ _____
8. $2-3/32 + 4-7/8 + 5-11/16 =$ _____
9. $2-3/8 - 1-23/32 =$ _____
10. Joan has a board that is $23-3/16$ " long. She needs a piece only $15-9/16$ " long. How much must be trimmed off? _____
11. What is half of $9/16$? _____
12. What is $3-5/8$ times $2-3/16$? _____
13. How many $1/32$ " in 5 "? _____
14. Don mixes $1-3/4$ cups of thinner into each gallon of stain he uses. How much thinner will Don need to thin $9-1/2$ gallons of stain? _____
15. A piece of wood $67-1/16$ " needs to be divided into 5 pieces of equal length. assuming the saw kerf to be $1/8$ ", how long will each piece be? _____
16. What is 5 divided by $3-3/4$? _____
17. Marie has to drill a $3/8$ " hole on center in a piece of wood $3-7/8$ " wide. What will be the measurement from each edge to the center of the hole? _____
from each edge to the edge of the hole? _____
18. How many feet in 56 "? _____
19. How many inches in $3-3/4$ '? _____
20. Use a tape to measure the dimensions of the given piece of wood. _____

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"*Computing with Fractions and Mixed Numbers*" Mary Mayfield, Instructor, Lane Community College, 1996.

"*Deciding When to Add, to Subtract, to Multiply or to Divide*" (This worksheet can be found in the Basic Math Curriculum) Mary Mayfield, Instructor, Lane Community College, 1996.

"*Developing Fraction Sense - Ruler Exercise*" David Shellabarger, Instructor, Lane Community College, 1995.

Developing Mathematical Thinking: A Complete Unit on Fractions with Blackline Masters and Teacher Lesson Plans, Jay Greenwood, 1986. The following worksheets were used.

"Sequence A - Lesson 4" (p. 17)

"Sequence A - Lesson 6" (p. 23) (as revised for tape measure fractions by Mary Mayfield)

"Sequence A - Lesson 2" (pp. 10-12)

"Sequence C - Lesson 2" (p. 75)

"Sequence D - Lesson 2" (p. 91)

"Sequence D - Lesson 6: Why do you think so?" (p. 99)

"Sequence G - Lesson 1" (p. 141)

"Sequence G - Lesson 2" (p. 145)

"*Equivalent Fractions*" Mary Mayfield, Instructor, Lane Community College, 1996.

"*Fraction Tile*" (page 54) from Aftermath IV, Seymour and others, Creative Publications, 1975. Used with permission.

"*Fraction Word Problem Practice*" Mary Mayfield, Instructor, Lane Community College, 1996.

"*Giant Inches*" (Eighths & Sixteenths...p. 1, Thirty-Seconds...p. 2) Mary Mayfield, Instructor, Lane Community College, 1996.

"*Giant Slalom*" Aftermath IV, pg. 100, Seymore and Others, Creative Publications, Worth, IL., 1975. Used With Permission.

Course Title: Fractions and Measurement in the Workplace

"Jumping Floozie" Problem Solving in Mathematics, Lane County Mathematics Project, Grade Level 4. pg. 79, Dale Seymour Publications, Palo Alto, CA, 1983. Used with permission.

"The Large Square Represents One Unit" Aftermath I, p. 81, Seymour and others, Creative Publications, Worth, IL, 1975. Used with permission.

"Line up with Fractions" Have Fun with Math, Louis Grant Brandes, p. 32, J. Weston Walch, Portland, ME, 1988.

"Measurement Practice - 1" Mary Mayfield, Instructor, Lane Community College, 1996.

"Measurement Practice - 2" David Shellabarger, Instructor, Lane Community College, 1995.

"Number Line" Mary Mayfield, Instructor, Lane Community College, 1996.

"Smallest Answer" Problem Solving in Mathematics, Grade Level 7, pp. 91 & 93, Lane County Mathematics Project. Dale Seymour Publications, Palo Alto, CA, 1983. Used with permission.

"Tape Measure Math" Mary Mayfield, Instructor, Lane Community College, 1995.

"Thinking about Fractions" Mary Mayfield, Instructor, Lane Community College, 1996.

"A Whole Thing ①" Aftermath I, p. 79, Seymour and others, Creative Publications, 1975. Used with permission.

"A Whole Thing ②" Aftermath IV, p. 52, Seymour and others, Creative Publications, 1975. Used with permission.

"A Whole Thing ③" Aftermath I, p. 23, Seymour and others, Creative Publications, 1975. Used with permission.

"Workplace Practice - 1" Mary Mayfield, Instructor, Lane Community College, 1996.

"Workplace Practice - 2" Mary Mayfield, Instructor, Lane Community College, 1996.

"Workplace Practice - 3" Mary Mayfield, Instructor, Lane Community College, 1996.

Workplace Training Project

Course Title: Fractions / Decimals/ Formulas in the Workplace

Course Description:

Content focuses on understanding the process of converting fractions to decimals and decimals to fractions, and calculating area and volume factors as used at the manufacturing company. Another goal is to teach participants how to work with $3/8$ " basis square footage because that is a standard calculation needed for the employer. Currently, employees use job aids to provide necessary production information. Participants use calculators in the training to arrive at totals and to check processes. Math problems participants need to use on the floor are a frequent subject in class whenever a participant or a supervisor brings them.

Student Outcome Objectives

Participants will demonstrate the ability to:

1. Demonstrate decimal sense on a number line and understand place value
2. Use addition, subtraction, multiplication and division of decimal numbers of the type commonly encountered on the job, including specific worksite applications of those concepts.
3. Demonstrate fraction sense on a ruler line and learn to compare fractions.
4. Add, subtract, multiply and divide fraction numbers of the type commonly encountered, including worksite applications.
5. Demonstrate the ability to convert fractions to decimals; and convert decimals to fractions.
6. Calculate area and volume.
7. Calculate area and volume conversion factors as used at the worksite, including a $3/8$ square footage basis.

Course Title: Fractions / Decimals / Formulas in the Workplace

Student Population:

Participants in this class are dryer tenders, assembly-line tenders and supervisors of a plywood mill. Most of these employees were required by the company to attend the training, but the employees themselves had asked for training in this area. All of the participants need the computational skills to fill in production sheets as a routine part of their work.

There have been several different classes with this title taught during the project. Each of them has had different teachers, and different participant populations. One class had 6 participants all testing at a basic skill level. Another class of 6 had several participants who scored almost 90% on the pre-test. Fortunately, the groupings were such that the class work could be tailored for the needs of the participants. Another class had 3 people, one of who had very low skills, and two with very high skills. Yet another class had 3 people with high skills.

Class Environment:

To facilitate attendance and make employees comfortable, all classes met on-site at the plywood mill. Training took place either in the non-smoking lunchroom in the mill, or in the conference room in the mill's office. The lunchroom has doors opening out to the manufacturing floor and to the outdoors. If the doors open during class, fork-lifts go by, and noise can be disruptive. There may be water on the floor if it is raining. For classroom props, the instructor uses a portable white board supported by an easel. The furniture is picnic tables and benches. The quiet, carpeted conference room has one long table and a built in white board.

Each of these classes takes place either before a worker begins his regular shift or after completing a full shift. One class began at 6:30 a.m., one at 8:45 a.m. one at 2:00 p.m., and one at 10:30 p.m. Each class was seven weeks long, meeting twice per week for one hour.

Instructional Techniques:

☐ Multi-media	☒ Small group	☒ Cooperative learning
☒ Lecture / demonstration	☐ Computer-aided	☒ Assignments
☒ Individualized instruction	☐ Simulation	☒ Peer tutoring
☐ Role-playing	☐ Project	☒ In-class practice

Fractions / Decimals/ Formulas in the Workplace

Syllabus of Course Activities:

I. Introductory Material

- Grant paperwork and pretest
- Review of objectives to be achieved in class and what they mean
- Add any additional objectives participants want covered in the class
- Preparation of individualized learning plan

II. Decimals

- Review of basic decimal concepts such as place value
- Practice in class with principles of computing with decimals and applying those principles generally and also as they apply in this workplace

III. Fractions

- Review of basic fraction concepts such as using fractions to name parts of things, the meaning of fractions, comparing fractions, etc.
- Review of mathematical operations using fractions
- Practice with word problems containing fractions

IV. Understanding the relationship between fractions and decimals and doing fraction/decimal conversions

V. Working with 3/8" Square Footage and Other Workplace Applications

- Further practice in making the transition between basic skills and how they are applied in the workplace.
- Practice using the concepts and applying them in work settings

VI. Concluding activities including grant paperwork, post-test and feedback

Activities I: Introductory Activities

1. Grant paperwork and pretest
2. Review of objectives to be achieved in class and what they mean
3. Add any additional objectives participants want covered in the class
4. Preparation of individualized learning plan

Materials and Resources needed:

- # "Factor Computation Pre-test" Instructors, Lane Community College, 1995.
- Grant paperwork
- Sample Objectives on Individual Learning Plan for class to review.

Description of Instruction

- Class completed grant paperwork.
- Instructor reviewed class objectives with the class, giving them an opportunity to add additional objectives.
- Class completed Individual Learning Plan
- Class took pretest

Performance Assessment Methods and Results:

- Completion of forms and individualized learning plans.
- The participants need to know what will be covered and that class will meet their needs. Reviewing the objectives and creating an ILP helps achieve those goals.

- A copy of the marked handout is in the **Handout** section of this curriculum.

Activity II: Decimals

1. Demonstrate decimal sense on a number line.
2. Add, subtract, multiply and divide decimal numbers.
3. Practical word problems using decimals.

Materials and Resources Needed:

- ## "*Number Line Decimals*" from Problem-Solving in Mathematics, Lane County Math Project, Grade 5, pp.197,199 Dale Seymour Publications. Palo Alto, CA, 1983 (Used with permission) ## (Copy of this included with Basic Math curriculum.)
 - ## "*Be Reasonable*" worksheets from Problem-Solving in Mathematics, Lane County Math Project, Grade 6, p. 179, Dale Seymour Publications . Palo Alto, CA, (Used with permission) ## (Copy of this included with Basic Math curriculum.)
 - ## "*Decimal Sense*" David Shellabarger, Instructor, Lane Community College, 1995. ## (Copy of this included with Basic math curriculum.)
 - ## "*Decimal Sense--Place Value*" David Shellabarger, 1995. ## (Copy of this included with Basic math curriculum.)
 - Number Power 2: Fractions, Decimals, and Percents, Jerry Howett, Contemporary Books, Inc., Chicago, IL 1988. (Appropriate pages from this book were used to review place value, addition, subtraction, multiplication, and division of decimals.)
 - Contemporary's Decimals, Math Exercises series. Robert P. Mitchell, Contemporary Books, Inc. , Chicago, IL, 1993.
 - ## "*Rules for Computing with Decimals*" Mary Mayfield, Instructor, Lane Community College, 1996. ## (Copy of this included with Basic Math curriculum.)
 - ## "*Decimal Review*" Mary Mayfield, 1996 ## (Copy of this included with Basic Math curriculum.)
 - ## "*Placing the Point I*" Calculate! Problem Solving with Calculators, Terrence G. Coburn Creative Publications, Mountain View, CA. 1988. ## (Copy of this worksheet can be found in the Basic Math Curriculum.) (Selected pages from this text used with permission)
 - "*Estimating Quotients*" Calculate!
- ## - A copy of the marked handout is in the **Handout** section of a different curriculum.

Course Title: Fractions / Decimals / Formulas in the Workplace

- “Placing the Point 2” Calculate!
- “Decimal Estimation 1” Calculate!
- ## “Decimal Estimation 2” Calculate! ## (Copy of this included in the Basic Math Curriculum.)
- ## “Hit the Target” Calculate! ## (Copy of this included in the Basic Math Curriculum.)
- ## “Secret Message Using Decimals” from Have Fun with Math, J. Weston Walch, 1988. for general review of decimals ## (Copy of this included with Basic Math curriculum)
- Appropriate pages from Contemporary’s GED Test 5: Mathematics or other source for participants needing more computation practice.

Description of Instruction:

1. Instructor drew a number line on the board and asked each group to fill in what they knew about place value, and more specifically using decimal places.

The more advanced classes used “*Number Line Decimals*,” “*Decimal Sense*,” and “*Decimal Sense--Place Value*” to improve their general understanding of decimals.

2. The group generated rules for adding decimals. One approach to this used the “*Rules for Computing with Decimals*” worksheet, that has a section for each operation (addition, subtraction, multiplication, division with decimals).
 - For each section, the first task was for the class, either individually or as a group, to estimate answers to the problems at the top of the page.
 - Next, participants used their calculators to get an exact answer.
 - Then, participants worked in pairs, using their general decimal sense, the information they had just generated, and whatever memory they had of the calculations, to generate the rules for doing that operation with decimals.
 - Finally, as a whole group, the class discussed and refined the rules, so everyone could write (and then have available for reference) a clear set of instructions.
 - The class also talked about which parts of the calculation would be easiest to forget or skip over, and made special note of these in the section of “*Rules for Computing with Decimals*” called “What to Watch For”.
 - In some cases participants did sample problems at the board.

Course Title: Fractions / Decimals / Formulas in the Workplace

3. For subtraction, multiplication and division of decimals the class used the same process.
4. The instructor used worksheets from Calculate! for estimation of decimal multiplication and division as seemed appropriate for the skills of the group and the time available.
5. The group brainstormed and listed applications for using decimals on the job, and the class worked through some sample problems.
6. The class used pages from Number Power 2 for further practice computing with decimals. Some of this practice was done in class, but participants completed most of this practice, including work-related word problems, on their own time. The results from practice done outside of class were checked at the next class meeting.
7. "Decimal Review" and "Secret Message Using Decimals" were used for more advanced review and practice.

Performance Assessment Methods and Results:

The teacher checked the accuracy of results of homework on decimal worksheets and returned them to participants.

Comments:

- Practice at home using decimal worksheets reinforced class practice. Those who did the practice improved noticeably faster than those who worked only in class. When teaching this next time, homework should be emphasized more at the beginning of the course.
- With more advanced groups, participants reviewed this subject by discussing the definition of a decimal, comparing decimals and reviewing decimal computation. This review took 30 minutes. How much time spent depends on the tested needs of the participants and on the skills demonstrated by participants during in-class discussions.

Activity III: Fractions

1. Demonstrate comprehension of fractions on a ruler line.
2. Add, subtract, multiply and divide fractions of numbers.
3. Practical word problems -- use of fractions (In a more advanced class)

Materials and Resources Needed:

- Calculators (preferably ones with fraction keys)
- Number Power 2, Jerry Howett, Contemporary Books Inc., Chicago, IL, 1988.
- Contemporary's Fractions, Math Exercises series, Robert P. Mitchell, Contemporary Books Inc., Chicago, IL 1993.
- Materials for "*Fraction Terms Matching Exercise*." This includes envelopes containing slips of paper, some with fraction terms written on them and some with examples of those terms on them.
- Customized problems using workplace skills and situations

Description of Instruction:

1. Warm-up review of fraction terms using "*Fraction Terms Matching Exercise*." The envelopes were given to small groups. Each envelope had separate slips of paper with fraction terms written on them, and slips of paper with examples of those terms on them. The task of each group is to match terms and examples. Then the groups check each other for discrepancies.

When matched pairs are agreed upon, the whole group defines each term on the worksheet and writes down the example from the matched pairs

2. Participants practiced comparing fractions using common sense and using common denominators to help make using fractions more comfortable.
3. The group discussed an addition of fraction (with like denominators) problem on the board, and then the group decided how to solve it. The term "reduce" was discussed and practiced, and several more problems were done on the board. The instructor distributed Number Power 2. Then participants worked on their own and checked their work with the group. There was a class discussion about applications for this skill on the job.
4. The class used a similar approach to subtraction, multiplication and division of fractions. Participants did appropriate pages from Number Power 2 as homework, as well as for practice during class.
5. The group listed applications for using each operation of fractions on the job as participants worked on them in class. The class worked through some sample problems.

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6. The class practiced their skills on a variety of workplace problems. Participants brought in some of these problems; supervisors and management contributed others. The instructors also went to participants' work areas to record situations that might require the math skills being taught.

Performance Assessment Methods and Results:

The instructor checked the accuracy of results of work on fractions practice and returned the papers to participants.

Comments:

- The participants with low skill levels found fractions much more difficult than decimals. One class spent 1 session on decimals, and 5 class sessions on fractions and their operations.
- Participants who needed it got lots of practice with worksheets on fractions. Participants learned to identify equal fractions and to compare fractions. Participants also learned to define a base unit in preparation for learning to work with 3/8" basis square footage.
- In a class with mixed abilities, the instructor used individualized instruction. One participant spent most of his time working with decimals and fractions and only a brief time on production square footage. The high-skill participants reviewed concepts, definitions and computational methods in half an hour, then spent most of an hour practicing with fractions, word problems.
- Participants enjoyed the opportunity to try out calculators with fraction functions. Most hadn't seen them before. Some went out and bought their own.

Activity IV: *Understanding the Relationship between Fractions and Decimals and Doing Fraction/Decimal Conversions*

1. Convert fractions to decimals
2. Convert decimals to fractions
3. Understand how to perform these conversions in workplace settings

Materials and Resources Needed:

- A worksheet developed by one of the instructors teaching the class. It lists fractions and decimals that participants would see at the manufacturing plant.
- Appropriate practice materials on fraction/decimal conversions.

Description of Instruction:

1. The class discussed the relationship between fractions and decimals. For example, $\frac{3}{4} = .75$. Participants discussed how many fraction/decimal equivalents they knew. After listing the familiar equivalents on the board, they were asked how to change a fraction to a decimal if you don't have it memorized.
2. The instructor worked through a number of sample problems on the board to demonstrate the process for changing a fraction to a decimal. Changing familiar fractions to decimals took the mystery out of the process.
3. Using the problems on a worksheet, participants practiced the process and checked answers with the calculator.
4. A similar procedure was used when converting decimals to fractions

Performance Assessment Methods and Results:

Participants could check their own understanding of the conversion process by using the calculator.

Comments:

- The calculators we used had fraction keys so it was easy to verify the conversions we were making. However, the calculators that participants used in the workplace did not have the fraction key, so they needed to understand how to make the conversion with pencil and paper or using a basic calculator.
- The advanced participants remembered how to make the conversions and reviewed this with only a brief discussion.

Activity V: *Working with 3/8" Square Footage and Other Workplace Applications*

1. Reviewed and practiced calculating area and volume
2. Practiced basic fraction/decimal skills and learned to apply these skills and understanding on the job.
3. Introduced and practiced calculating area and volume conversion factors as used at the mill, especially SF 3/8 basis (3/8" square footage basis.
4. Reviewed using percentages as they relate to this workplace environment.

Materials and Resources Needed:

- Calculators (preferably ones with fraction keys)
- Appropriate practice materials on solving area and volume problems
- # "*Volume Applications*" David Shellabarger, Instructor, Lane Community College, 1996.
- # "*Calculating SF_{3/8} Basis Factors and SF_{3/8} Basis Production*" David Shellabarger, 1996, that provides clear instructions for conversion.
- Other customized problems using workplace skills and situations
- # "*Practice with SF(3/8 Basis) Factors and Production*" Mary Mayfield, Instructor, Lane Community College, 1996. .
- # "*More 3/8 Basis*" Mary Mayfield, 1996.

Description of Instruction:

Area and Volume

1. The terms area and volume were discussed and then formulas were written on the board. Participants matched the formula with its use (area of triangle, circumference of circle, etc.) and gave the reason why they connected the two.
2. The class used examples from the workplace to practice using the formulas i.e. plywood sheets, glue tank, number of plywood sheets loaded on a truck, etc.
3. Worksheets, including "*Volume Applications*," reinforced skills and offered practice on the skills learned.

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Calculating Material Usage with a $\frac{3}{8}$ Production Basis

- The manufacturing plant uses a $\frac{3}{8}$ " basis for calculating usage and production of their materials. See "Comments" for a further explanation.
- 1. The instructors talked briefly about the theory and usefulness of working with $\frac{3}{8}$. The classes used "*Calculating $\frac{3}{8}$ Basis Factors and $\frac{3}{8}$ Basis Production*" worksheet to discuss and provide participants practice with this skill. It was important that both fraction and decimal methods of converting to a $\frac{3}{8}$ " basis were clear, so the class practiced with several common fraction and decimals used in the manufacturing plant to help participants to clearly see the relationship between them. First participants worked the fractions, then took the same information and did it in decimals. They put all of the above information into a grid to show how the information corresponded and in fact gave them the same answers in different forms. * Note: This last step wasn't necessary in high-skill groups but was with lower-skill participants.
- 2. Instructors took the participants into the plant and had them measure stacks of veneer and convert these actual measurements to $\frac{3}{8}$.
- 3. The class then worked on word problems directly related to plant operations and that offered enough practice to get them truly comfortable with this skill.
- 4. Reviewed much of the information covered during the course to help cement the skills in participant's minds for those participants with lower skill levels.
- 5. Worked with percentages as they relate to a production environment. The employers requested this skill be included in the curriculum.
- 6. The class that mastered understanding and working with $\frac{3}{8}$ requested some work with algebra and geometry. Participants used geometry to figure more complicated area and volume problems. They also study different types of geometrical figures and the Pythagorean theorem. In class discussion, participants and the instructor tried to relate this information to situations that might occur in the plant.

Performance Assessment Methods and Results:

- Using customized worksheets with problems from the workplace gave both instructor and participants a good idea of how well they might be able to apply these skills.
- Instructors made sure that all participants knew how to successfully complete these problems.
- Most of the assessment taking place throughout every class session was individual and immediate. The instructor watched participants as they worked, asked questions, provided feedback or additional information. If there were significant problems, they became the focus of a group discussion.

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- The instructor noticed the kinds of mistakes participants made, and how quickly they were able to move through the practice exercises. The degree of success being experienced strongly influenced the speed with which the class moved
- Participants often took practice materials home to work on them, and the instructor used these to assess their level of understanding.

Comments:

- For those unfamiliar with $3/8''$ basis square footage ($SF_{3/8}$), it is a method of measuring plywood. A 4 x 8 sheet of plywood $3/8''$ thick is used as the standard and thus is 32 square feet, $3/8''$ basis. A 4 x 8 sheet $3/4''$ thick is equivalent to 2 of the standard sheets and is therefore 64 square feet, $3/8''$ basis. A sheet $1/8''$ thick is equivalent to $1/3$ of the standard sheet or 10.667 square feet, $3/8''$ basis. This method is also applied to stacks of veneer or plywood. For instance, a 4-foot stack of veneer is ~~theoretically~~ equivalent to 128 pieces of $3/8''$ plywood, or 4096 (128×32) square feet $3/8''$ basis. In actuality, air space in the stack and veneer that is not perfectly flat affect these equivalents and formulas are adjusted accordingly.
- The production sheet conversion tips worked extremely well for those well grounded in mathematics. For those with low basic skill levels, another level of explanation preceded the use of the sheet. The tips sheet relied on participants understanding of how applying formulas works. Participants with low skills levels needed to be led into that process more slowly, although they did learn how to understand and make use of the conversions required in their work setting.
- Working with the production sheets was one of those "AHA" experiences for the some participants. They could quickly see the benefit in their daily work lives of having learned the fraction/decimal conversions.
- Instructor spent approximately 4 class sessions of $SF_{3/8}$. The theory discussion took only a few minutes; the rest of the time is seeing how the theory is applied, asking questions, then going a little deeper, and doing a few more complicated problems.
- Participants often helped the instructor understand the areas they need to improve their skills. Particularly in the areas of algebra and geometry, participants and instructor had to work together to shape what was taught, so that it applied to what was needed.

Activity VI: Completion Activities

1. Complete grant paperwork
2. Give post-test to participants
3. Provide feedback to participants, and get feedback from participants on class.

Materials and Resources Needed:

- Grant forms
- "*Factor Computation Post-Test*" (which was similar to # # "*Factor Computation Pre-Test*" # #, a copy of which is included with the Basic Math curriculum.)
- Individualized Learning Plans (with Part I already completed)

Description of Instruction:

1. Participants completed the paperwork for the grant
2. Participants took the post-test
3. Participants completed the Individualized Learning Plans.

Performance Assessment Methods and Results:

- Participants reviewed the progress they had made between the pre and post test results.
- Participants also had a chance to provide input to the instructor on the quality of the class.
- One teacher wrote individual notes to each participant giving them some idea of their progress because time ran out in the last session.

Comments:

- During the last class, the instructors would have liked to have at least an hour and a half because there was not enough time for interaction and wrap-up.

Course Summary

Background:

The company wanted dryer tenders, line tenders, and supervisors to better understand the following math processes: converting fractions to decimals and decimals to fractions, measurement, surface area, and square footage. These co-workers use these concepts in daily paper work but use calculators to arrive at totals. The purpose of the class was to help co-workers better understand the concepts involved with their paper work so there would be fewer errors.

Expectations:

Company:

The company required all lead co-workers to participate in the fraction/decimal conversion class in order to better understand their daily paper work. Though not a requirement, other interested co-workers could attend the class.

Learner:

Because this was a required class for co-workers, some participants seemed apprehensive at first. Once class started, participants seemed to relax and felt comfortable asking questions and clarifying information.

Instructor:

The three instructors who taught these classes each brought different expertise to the situation: math experience, construction experience, learning disabilities background. They found that sharing ideas and teaching approaches proved helpful, especially at the beginning of the class.

Evaluations:

Company:

The company expressed satisfaction with the training, especially with attendance and completion rates. The Site Advisory Team (SAT) members wanted to track how well participants retained math concepts six months after they had completed. SAT members tried to gather surveys along with some math related questions from participants; however, a heavy production time limited the amount of data collected.

Learner:

Scores on post tests showed an average 27% gain. Participants rated the class above satisfactory. Comments included how much the training helped participants review math

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concepts that they had forgotten, and how they learned math related to the job. One man who was to retire in three months, was, nevertheless, required to attend. He came to every class and did all of his homework. The day of the post test he was on vacation, but he came in to take the test. He felt particularly proud of his accomplishment in completing the class and felt as if he learned a lot.

Instructor:

Three different classes were taught by three different instructors in order to meet shift time schedules. Each class had its own personality, interests and skill level. There seemed to be a good match among instructors and classes. Each instructor felt that participants progressed well given participants' skill level. Despite the fact that the training was required of participants, participants really wanted to learn. Coming up with the right answers was not the point, learning the process of how you get the right answer was important to them.

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HANDOUTS

Factor Computation Pre-test

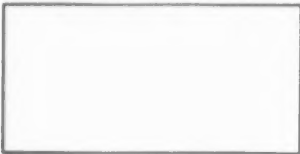
Name _____

Work by hand. No calculators.

1. Multiply: $\begin{array}{r} 395 \\ \times 76 \\ \hline \end{array}$	2. Mark 1.3 on this scale $\begin{array}{c} + \text{-----} + \\ 0 \qquad \qquad \qquad 2 \end{array}$
3. Add 1.35, 12.7, 5.96	4. Subtract .0076 from 1.2000
5. Multiply 3.09 by .048	6. Convert $\frac{1}{6}$ to a decimal number
7. Add $\frac{7}{8}$ and $\frac{3}{8}$	8. Divide $\frac{7}{8}$ by $\frac{3}{8}$
9. Divide $32 \overline{)784}$	10. Put in order: $\frac{7}{10}, \frac{3}{10}, \frac{3}{8}$

Factor Computation Pre-test

Name _____

11. Add: $\begin{array}{r} 3\frac{3}{4} \\ +2\frac{1}{6} \\ \hline \end{array}$	12. Find the area this figure in sq. inches: 96"  54"
13. Find the volume of a cord of wood (4 feet wide, 4 feet high and 8 feet long).	14. 28 sheets of 1/7 inch veneer are stacked. Calculate the height of the stack if all pieces were to fit tightly without gaps.
15. 1200 sheets of 1/2" plywood are produced. Find the production on a 3/8" basis.	

Volume Applications

Use the formulas on the handouts in working the following examples.

Tank Volume

- 1a. List the shape and dimensions of your glue storage.
- b. Calculate the tank volume in cubic feet.
- c. Calculate the capacity in gallons. Use 1 cu. ft. = 7.49 gallons

Log Volume

- 2a. Find the scaling cylinder volume of a log which tapers from 8 in. diam. to 4 in. diam. and is 16 ft. long.

- b. Find a more realistic volume of the same log treating the log as the frustum of a cone.

$$V_{\text{frustum of a cone}} = \frac{1}{3} \pi h (R^2 + r^2 + Rr)$$

Log Deck Volume

The saw log deck shown has the following visually squared dimensions:

length 150 ft.
half at height 32 ft. half at height 26 ft.
width 20 ft.

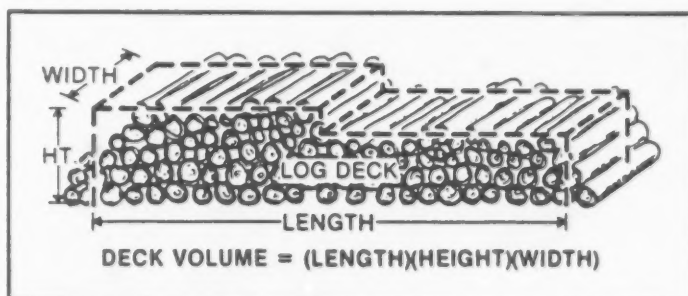


Figure 4 — Cubic volume of the deck is determined by measuring height, length, and width of a visually squared deck. This volume includes bark and air and is converted to scaled volume, either BF or CF, by multiplying by a deck factor.

- 3a. Calculate the gross deck volume.
- b. Calculate the inventory volume in board feet using 2.83 bf per cf

Volume Applications

Board Feet Per Cubic Foot

One formula for calculating net bf per cf (West side Scribner scale rules) is

$$\text{bf/cf} = 10.16 - 0.04 \times D - 88.18 \times (L/D) + 290.58 \times (L/D^2)$$

4a. Calculate the bf per cf conversion factor for a 6 inch log

b. Calculate the bf per cf conversion factor for a 12 inch log

Veneer Volume

One formula for converting cubic feet of log volume to veneer volume (untrimmed green veneer) is:

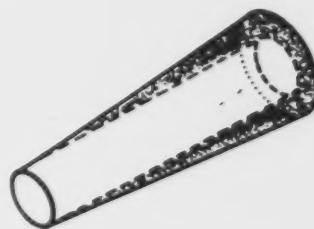
$$\text{Veneer volume cf} = -3.493 \times (\text{number of logs}) + 0.8004 (\text{log volume cf})$$

5a. Find the "before loss" veneer volume in cf for the deck in the example above.

b. Find the 3/8 basis square feet of veneer production for the deck in the example above. Use a factor of 18% loss due to shrinkage, lay-up, and panel trim. Recall 1 cu. ft. = 32 sq ft. 3/8 basis.

Log Volume

6a. Find the scaling cylinder volume of a log which tapers from 8 in. diam. to 4 in. diam. and is 16 ft. long.



b. Find a more realistic volume of the same log treating the log as the frustum of a cone.

$$V_{\text{frustum of a cone}} = \frac{1}{3} \pi h (R^2 + r^2 + Rr)$$

Calculating SF3/8 Basis Factors and SF3/8 Basis Production

Method

32 square feet of 3/8" veneer is the same amount of wood as a sheet of 3/8 inch plywood. Consider one 3/8 veneer as one 3/8 sheet of plywood, so the 3/8 sheet of veneer is 32 SF on 3/8 basis.

Now consider the case of 1/8 veneer. 32 square feet of 1/8 veneer is 1/3 of a 3/8" sheet. This can

be gotten by performing the calculation $\frac{\frac{1}{8}}{\frac{3}{8}} = \frac{1}{3}$.

$\frac{1}{3} \times 32 = 10.666$ square feet of 3/8 plywood per sheet of 1/8 veneer. 10.666 is the SF3/8 factor for 1/8 inch veneer. In essence, each sheet of 1/8" veneer is 1/3 of a sheet of 3/8 inch plywood.

The same method can be done with decimals. Converting the fractions to decimals, $1/8 = .125$ and $3/8 = .375$.

$\frac{.125}{.375} = .3333$, and $.3333 \times 32 = 10.666$, as before.

Examples

Example 1) Find the production of SF3/8 basis for 900 sheets of 1/8 inch veneer 54" x 96".

$$900 \times 10.666 = 9599 \text{ SF3/8.}$$

Example 2) Find the production of SF3/8 basis for 15,000 square feet of 1/8 inch veneer 54" x 96".

$$15,000 \times 1/3 = 5,000 \text{ SF3/8, since } 1/8 \text{ is } 1/3 \text{ of } 3/8$$

Example 3) Find the SF3/8 factor for sheets of 1/10 inch veneer 54" x 96".

$$\frac{\frac{1}{10}}{\frac{3}{8}} = \frac{1}{10} \times \frac{8}{3} = \frac{8}{30} = .2666. \text{ This is .2666 SF3/8 per SF 1/10.}$$

The factor for sheets is therefore $.266 \times 32 = 8.533$ SF3/8 per 32 SF1/10. This is the same as 8.533 SF3/8 per sheet of 1/10 veneer.

Example 4) 2500 sheets of 1/10 veneer are produced. Find the production on SF3/8 basis.

$$2500 \times 8.533 = 21,333 \text{ SF.}$$

Using units, we can write $2500 \text{ sheets} \times 8.533 \text{ SF3/8 per sheet} = 21,333 \text{ SF.}$

Practice with SF(3/8 Basis) Factors and Production

Previously, we learned to figure the factor (3/8 basis) for a particular size veneer by considering the ratio of the veneer we're working with to 3/8", then dividing the thickness of our chosen veneer by 3/8. (For instance, for 1/8" veneer, 1/8 divided by 3/8 gave us a factor of 1/3.)

We then multiplied that factor by 32 (the number of square feet in a 4' x 8' piece of plywood) to obtain the square footage factor (3/8 basis)--that is, the number of square feet of 3/8" plywood that would be contained in one sheet of our selected sheet of veneer. (For instance, for 1/8" veneer, $1/3 \times 32$ sq. ft. gives us a square footage factor (3/8 basis) of 10.6666 or, properly rounded, 10.667.)

Sometimes, though, instead of needing to know the how the square footage of a particular product compares with a 3/8" sheet of plywood, we might need only to know how many sheets of 3/8" plywood a given number of sheets of our product would be equivalent to. To do this, we go back to our original factor and multiply that factor by the number of sheets we have of our product. (For instance, if we had 900 sheets of 1/8" veneer, we could multiply that 900 by our original factor of 1/3 to obtain 300--that is, 900 sheets of 1/8" veneer are equivalent to 300 sheets of 3/8" plywood.)

Now for some practice.

- 1) If such a thing as 1/4" veneer were being produced, what would be the factor (3/8 basis) for it?

- 2) If you had 1200 sheets of the 1/4" veneer, what would be the equivalent number of sheets 3/8 basis?

- 3) Figure the square footage factor (3/8 basis)--abbreviated SF(3/8) factor--for standard size sheets of 1/4" veneer.

- 4) Now use this SF(3/8) factor to find the square footage (3/8 basis) of your 1200 sheets of 1/4" veneer.

5) If such a thing as $1/9''$ veneer were being produced, what would be the factor for it?

6) If you had 3000 sheets of this $1/9''$ veneer, what would be the equivalent number of sheets $3/8$ basis?

7) What would be the $SF(3/8)$ factor for $1/9''$ veneer?

8) What would be the $SF(3/8)$ of your 3000 sheets of $1/9''$ veneer?

This process can be applied to plywood as well as veneer.

9) Find the production of $SF(3/8)$ basis for 2500 sheets of $7/8''$ plywood.

10) Find the $(3/8)$ factor for sheets of $7/8''$ plywood $54'' \times 96''$; then figure the equivalent number of sheets $(3/8)$ basis for the 2500 sheets of this plywood.

11) Find the $(3/8)$ factor for sheets of $1-1/4''$ plywood $54'' \times 96''$; then figure the equivalent number of sheets $(3/8)$ basis for 2500 sheets of this plywood.

12) Find the production of $SF(3/8)$ basis for 2500 sheets of $1-1/4''$ plywood $54'' \times 96''$.

More 3/8 basis

1. Calculate the SF(3/8) for 5/8" plywood.
2. Calculate the square footage of 1800 sheets of 5/8" plywood on 3/8 basis.
3. What would be the production (in sheets of plywood, 3/8 basis) of 2200 sheets of 5/8" plywood?
4. Calculate the factor that would be used to convert information on 1" plywood to 3/8 basis equivalent.
5. Calculate the SF(3/8) factor for 1" plywood.
6. This factor would also be the factor for an inch stack of perfect veneer. However, in a real stack of real veneer, there are conditions that affect and change the number of sheets of veneer in that inch. What are some of those factors?
7. If a 4' stack of 1/10" veneer actually contains 384 sheets of veneer, how many sheets per inch is that?
8. To calculate the SF (3/8) factor for the above veneer, consider the ratio of the number of actual sheets per inch to the number of sheets that would be in a perfect inch of 1/10" veneer (this ratio is 8-1/2:10 or 17:20); then calculate the SF(3/8) factor as we've done before (that is, by dividing the actual ratio by 3/8; then multiplying by 32).
9. How does this factor compare to the factor actually used to figure square footage (3/8 basis) for inches of 1/10" veneer?
10. If you were using the factor 73.5545 to figure SF (3/8 basis) for inches of 1/10" veneer, how many sheets of veneer are being presumed to be in an actual stack of 1/10" veneer?

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Workplace Training Project

Course Title: Using Calculators in the Workplace

Course Description:

The primary focus of this course is to familiarize participants with four-function calculators with 3 or 4-key memory. Participants have an opportunity to learn about and practice using the addition, subtraction, multiplication and division constants. They learn to use the calculator to figure percents, powers of numbers, square roots, and making fraction/decimal conversions. They also learn to use the memory function of the calculator. They learn about calculator logic and how to work with large numbers on the calculator. A secondary part of the class is to offer participants a brief introduction into other types of calculators (scientific, graphing, financial). This information gives participants some idea of the range of possibilities and helps them understand whether they need a more sophisticated calculator for their daily work tasks.

The instructor gave participants a choice of objectives for each class. As a result, the instructor tailored each class to meet the requested objectives as well as some additional objectives chosen by the instructor. Later the class covered some additional concepts that participants also needed, but that were new to them. One of the instructor's objectives was that participants learn to do calculations efficiently. The instructor also emphasized the importance of participants continuously double checking to see that the process they used was appropriate for the problem at hand, and that their answers were reasonable.

Student Outcome Objectives

Participants will demonstrate the ability to:

1. Perform basic calculations more efficiently.
2. Perform calculations involving constants and memory.
3. Perform fraction, decimal, and percent equivalents and computations on the calculator.
4. Learn to use the calculator to make computations involving exponents and square roots.
5. Learn to work with large numbers on the calculator and understand what happens when answers to calculations are too big for the calculator display.

Course Title: Using Calculators in the Workplace

Student Outcome Objectives (continued)

Participants will demonstrate the ability to:

6. Experiment with or observe the use of more sophisticated calculators (scientific, financial, graphing) and understand the logic and conventions of typical calculators well enough to use different calculators.
7. Understand enough of the basic capabilities of more sophisticated calculators to know whether they need to consider using such a calculator.

Student Population:

The class had four participants, self-selected by interest in the topic. One person was office staff, one worked in shipping, one worked in maintenance, and one worked in production.

Another class of six participants all held different roles in the shipping department.

Class Environment:

The class met in the company conference room that, though a bit large for this class, has comfortable seating at tables and is equipped with an overhead projector, white board, etc. The class was pretty informal, with participants asking questions, working together, etc., and advancing to new material as everyone seemed comfortable with previous material.

The class consisted of six 1-1/4 hour sessions. Ideally the class should be 8 to 10 sessions to allow for more practice in class. Participants attended after working hours.

Instructional Techniques: ✓

☐ Multi-media	✓ Small group	✓ Cooperative learning
✓ Lecture	☐ Computer-aided	✓ Assignments
✓ Individualized instruction	☐ Simulation	☐ Peer tutoring
☐ Role-playing	☐ Project	✓ In-class practice
Other:		

Using Calculators in the Workplace

Syllabus of Course Activities:

- I. Introductory activities
- II. Basic operations on a calculator
- III. Working with calculation constants
- IV. Estimation and deciding when to use a calculator--rather than doing calculations in your head or with pencil and paper
- V. Other operations on the calculator--fractions, decimals, percents, powers, square root
- VI. Understanding and using the calculator's memory functions
- VII. Large numbers and what happens when answers are too large for the calculator display
- VIII. General introduction to different types of calculators
- IX. Concluding activities

Activity I: <i>Introductory Activities</i>
--

1. Grant paperwork
2. Review of objectives to be achieved in class and what they mean
3. Preparation of individualized learning plan for each participant
4. Taking of pretest indicating knowledge of calculators and basic math operations

Resources and Materials needed:

- Grant paperwork
- Sample Objectives on the Individual Learning Plan for class to review.
- # "Calculator Pre-test"
- # "*Learning Log for Calculator Training*" Mary Mayfield, Instructor, Lane Community College, 1996.

Description of Instruction:

1. Instructor helped participants complete all the preliminary paperwork.
2. Participants took the "Calculator Pre-test."
3. The instructor explained how to use the "*Learning Log for Calculator Training*." This log can be a useful tool to encourage participants to practice with the calculator and think about what they learned and how to apply it. At each class session participants were to note under "Concept Learned" what was covered in that session, and in the following weeks make note of where they used the concepts in their lives.

Performance Assessment Methods and Results:

- The participants need to know the subjects covered and that this class will meet their needs. Reviewing the objectives and creating an Individualized Learning Plan achieves that goal with the participants.
- The "*Learning Log for Calculator Training*" seemed to be a good idea, but participants didn't really use it. Because of limited in-class time, the instructor did not put much emphasis on making this a useful tool.

- A copy of the marked handout is in the **Handout** section of this curriculum.

Activity II: Calculator Basics

1. Activities and discussion to review or establish basic familiarity with the calculator.
2. Instructor checked each participant's ability to perform basic operations on the calculator--adding, subtracting, multiplying, dividing--using the correct buttons and coming up with the correct answers.

Resources and Materials Needed:

- A standard 4-Function Calculator for each participant

Description of Instruction:

1. The instructor went over basic calculator information with the participants in a discussion format, asking questions like the following:
 - "Look at the calculator and tell me how many digit keys there are."
 - "How many operation keys are there? "
 - "How many digits will appear on your calculator's display? "
 - "What's the largest number you can display? "
 - "What's the smallest number you can display? "
 - "What key will you press to erase your calculator's display?"
 - "How are negative numbers displayed on your calculator? "
 - "How are the memory keys on your calculator labeled?"
 - "Enter (What does this mean?) $97 + 15$ (no equals). Whoops you actually meant to add 18. What do you do?"
 - "How do you cancel the entire calculation? What do you think AC or CA means?"
2. The class experimented with what happens when you push a number on a calculator, push an operation and then push the equals button. When you push equals again, participants noted what happens to the total. The class looked at what happened with the different operations and discussed how this might be useful on the job.

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Performance Assessment Methods and Results:

- The instructor paid attention to responses to questions and to what participants were doing with their calculators to be sure that everyone was following the instruction.

Comments:

- The opening activity (discussion and hands-on to gain familiarity with calculator) worked well. Participants seemed both involved and comfortable. They asked questions, made comments and stayed with the activity.
- One class spent very little time on basic calculations because everyone knew how to do them, so they breezed through it very quickly. Another class spent more time working on these basic calculations as participants were neither accurate nor efficient with them.

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Activity III: Working with Calculator Constants

1. Worked with calculation constants, beginning with an addition constant
2. When addition constants were clear, the class learned about constants for subtraction multiplication and division

Resources and Materials Needed:

- # "Checking out the Addition Constant " (p. 3), Calculate! Problem Solving with Calculators (Whole Number, Decimals, Percents) by Terrence G. Coburn, Creative Publications, Mountain View, CA. 1988.
- "Checking Out the Subtraction Constant" (p. 4), Calculate!
- "Checking Out the Multiplication Constant"(p. 7), Calculate!
- "Checking Out the Division Constant" (p. 8), Calculate!
- # "Practice With the Constant Feature of Your Calculator" Mary Mayfield, Instructor, Lane Community College, 1996.
- A standard 4-Function Calculator for each participant and the instructions that came with it

Description of Instruction:

1. The instructor introduced the concept of a calculator constant and showed them how it worked when adding. The class used "Checking Out the Addition Constant " to understand the addition constant and to practice using it.
2. Once the group understood the concept of an addition constant, the instructor had them check the calculator directions for the addition constant. This was to prepare them for reading and understanding calculator use instructions, which can be rather minimal. Most of the calculators had been bought in bulk and did not come with instructions. The instructor copied the instructions from the box of one that had been bought singly, so that each participant had a copy. The copy provided a place for them to make notes about the functions or the directions.

The participants then used the calculator instructions to try to figure out how the subtraction constant worked. Because the instructions included with the calculator are short and cryptic, understanding the addition constant first made it a little easier to use the calculator's directions to understand the other constants. After they worked with the calculator's subtraction instructions, the instructor handed out a worksheet from Calculate! titled, "Checking Out the Subtraction Constant" to give further instruction and practice on using the subtraction constant. The class used the same methods to practice working on multiplication and division constants.

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3. Used "*Practice With the Constant Feature of Your Calculator*" to give participants practice on possible situations they might run into on their job. The worksheet gives them a place to answer each question and also a place to write out the keystrokes they used to come up with the answer.
4. The group talked about places in the workplace where these constants might be a useful tool. One participant suggested that since they assume all crates for pipe weigh the same, they could use the subtraction constant to subtract the crate weight off the weight of an order of pipe.

Performance Assessment Methods and Results:

- The instructor could see if participants were following the instruction:
 - By whether or not they could do the procedures being discussed
 - By the answers they obtained on their calculators
 - By the questions they asked
- The instructor also wandered the room and watched what participants were doing on the worksheets.

Comments:

- While teaching this class, the instructor had to remind the participants frequently that the point is to learn how to most efficiently learn to use the calculator. The goal is not necessarily getting the *answer*, but *how did you get to the answer*. The instructor taught participants a way to write down each keystroke. This was helpful because they are not used to thinking about how they arrive at an answer on a calculator. The worksheet "*Practice With the Constant Feature of Your Calculator*" was just about enough practice and was a good introduction to learning how to work efficiently with the calculator.
- Designing the worksheets with a place for the "answer" and a place for each key stroke worked well. Originally the place for keystrokes was a series of short spaces, asking participants to use one space per keystroke. Despite instructions, the participants did not use those spaces as designed. So, even though the instructor preferred that format as the clearest way to note the keystrokes, she modified the worksheets. Instead, they just have a line where participants write the keystrokes they used. Teaching the other would consume more time and energy (and possibly create more frustration) than it was worth.
- Some participants felt the "constant" feature was just a waste of time and was not useful. The instructor did not have enough understanding of their actual job tasks to determine whether there were times that this could be a useful feature at that worksite or not. It seemed as if the constant feature could be very helpful, but it was a hard point to argue.

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Activity IV: Estimation and deciding when to use a calculator

1. Work with mental math versus using a calculator and gain experience in determining when to estimate and when to use the calculator.
2. Use mental math and number sense to recognize reasonable (and unreasonable) answers to math problems.

Resources and Materials Needed:

- Selected computation problems for Calculation Competition
- Calculator Power: The Complete Handbook by Robert Mitchell, Contemporary Books, Chicago, IL, 1991 pp. 13, 15, and 17.

Description of Instruction:

1. Used an exercise called Calculation Competition. The class divided into 3 groups. The instructor put a variety of computations from simple to fairly difficult (one at a time) on the overhead projector. One group used pencil and paper; one used mental math; one used calculators. The group rotated the methods twice so each person used each method. The instructor tallied the first correct answer on the board under each method. Participants could only call out the answer they solved by the method assigned to their group.

The objective of the activity was to stimulate thinking and discussion of when a calculator is the appropriate computation choice. A further objective was to have participants recognize that they can and do perform mental math. The class briefly discussed some mental math techniques. If time allowed, it would be useful to revise and expand this activity to include practice with a variety of mental math computations.

2. Participants individually did the 24 computations on p. 13 of Calculator Power. For each problem, they chose whether to do it in their head, use pencil and paper, or use their calculator; then they tallied how many they did each way. This was another opportunity for people to see that they can often do calculations more quickly in their heads than they could press all the buttons required.
3. The group worked on estimating answers and arriving at a reasonable answer before doing actual calculations on the calculator. Then they would know whether or not their calculator result was a reasonable answer. The class touched on estimation several times, and in different ways, because it is an important part of understanding what you are doing and knowing whether or not your answers are accurate.
4. Participants individually did Section C, p. 15 and Section B, p. 17 from Calculator Power. These were addition and subtraction problems with their answers, some of which are wrong. The task was to use estimation and number sense to discover the incorrect ones.

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Comments:

- Page 13 of Calculator Power involves only addition. It would be more effective to have an exercise like this that included all four basic operations. The work in #4 above would also have been more effective if it had included all four operations.
- The company wanted two strong emphases:
 1. Being able to estimate a reasonable answer
 2. Double checking answers for reasonablenessThey felt that many of the computation mistakes that their employees make could be corrected "in the moment" if they had and practiced this skill.
- One class did not do these activities because of lack of time. It would have been a useful addition to the class, and would be addressed if there were more sessions. More time and emphasis on learning, practicing and using estimation for situations where an estimate is a "good enough" answer would be helpful in many situations. Estimation serves as check on the "sense" of a calculator or pencil and paper answer.

The instructor's experience in this setting and elsewhere is that people often do not understand the skill or value of estimation and therefore make avoidable mistakes. They also spend extra time calculating an exact answer that is not necessary, or become frustrated with the hassle and their own lack of skill.

- The "mental math" section was really brief; it would be good to be able to spend more time on this.
- This was a good way to practice using the calculator and using basic skills.

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Activity V: Operations on the Calculator such as Fractions, Decimals, Percents, Powers, and Square Root

1. Reviewed how to do fraction, decimal, and percent calculations without a calculator. (It was a very quick run-through as this was only a 6-session class.)
2. Reviewed the instructions that came with a calculator describing how to perform these operations on this calculator.

Resources and Materials Needed:

- # "*Changing Fractions to Decimals*" (p. 53), *Calculate! Problem Solving with Calculators* by Terrence G. Coburn, Creative Publications, Mountain View, CA. 1988
- # # "*Percent Means Per Hundred*," an information sheet written by Mary Mayfield, 1996.
(Copy of this page included in the Basic Math Curriculum.)
- "*Any Which Way*" (p.66), *Calculate!*
- "*Percent of Increase or Decrease*" (p.75), *Calculate!*
- # "*Percent Problems*" (p.81), *Calculate!*
- # "*Compounding Things*" (p.82), *Calculate!*
- *Calculator Power: The Complete Handbook* by Robert Mitchell, Contemporary Books, Chicago, IL. 1991
- Instructions for the calculator
- Scientific Calculator

Description of Instruction:

1. The class discussed how to use a calculator to work with fractions. The calculators being used in class did not have a fraction function. If participants did need to use their calculators to compute with fractions they had to convert the fractions to decimals and work with them in that form. The instructor did, however, bring a scientific calculator that had fraction functions to class to show how to compute fractions on such a calculator.
2. The class reviewed converting fractions to decimals using pencil and paper, and then did the same process on the calculator. For practice, participants worked among themselves to find the decimal equivalents of fractions the instructor had written on the dry erase board; then, they did the worksheet "*Changing Fractions to Decimals*." Finally, the instructor asked them to interpret the calculator directions for " $1/8 = 0.125$." This procedure gives the decimal equivalent for a unit fraction. Participants discussed how to extend this to other fractions (that is, first find the decimal equivalent for the unit fraction, then multiply that answer by the numerator of the fraction you are trying to change).

- A copy of the marked handout is in the **Handout** section of a different curriculum.

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3. The calculator has no exponent key, but you can compute 4^2 by keying $4 \times =$ or compute 4^5 by keying $4 \times = = = =$. You can build on that to calculate other powers. The class worked some with that process, but the instruction was very limited. The participants do not work with powers on the job and did not feel they needed more information.
4. Percents. The instructor emphasized understanding that percent means how many out of 100 and the class did some mental math finding 1%, 10%, 50%, 100%, etc. of some numbers. Gave the information sheet "*Percent Means Per Hundred*" to participants as a reference and used it as a way to illustrate and review this concept.

Participants read the calculator instructions to determine how to find percentages on the calculator and someone verbalized these instructions for the class. After a few practice problems, participants did the worksheet "*Any Which Way*," that gave practice using the calculator for percentages. It also illustrated the concept that " $x\%$ of $y = y\%$ of x " (for example, 30% of 100 is the same as 100% of 30). The class talked about different sets of keystrokes for these operations and which ones were most workable, most efficient, and easiest to remember. After doing a few of the problems in class, participants took it home for further practice.

Participants read the calculator instructions for computing

" $200 + (200 \times 10\%) = 220$ " and for computing " $500 - (500 \times 20\%) = 400$ "

Then each participant wrote a set of directions for increasing and for decreasing a given amount by a given percent.

In pairs, they read each other's instructions and used them to actually do some other problems written on the board by the instructor. This gave everyone a chance to thoughtfully look at the directions and to practice clearly verbalizing what they know. (This was hard for many people.)

The class reviewed how to find the percent when you know the part and the whole; then the class did this on the calculator.

The class worked some with percentages that participants use on the job. One problem often encountered at the plant is percent of overweight, which is similar to a percent of increase or a percent of decrease. Class worked on and discussed "*Percent of Increase or Decrease*," then practiced a few overweight/underweight problems.

Those participants who had completed the rest of the work also worked on "*Percent Problems*."

5. In the last class, the class worked with other practical applications of percentage using "*Compounding Things*." It shows how to use the calculator to figure balances on amounts earning interest compounded yearly. In addition to the worksheet, the class calculated the balances at the end of 5 years of \$500 compounded annually at 6% interest and at 10% interest; then they compared these balances. Some participants did additional variations on this theme. There was also some class discussion about the fact that most interest these days compounds much more frequently than annually, and what effect that would have on balances, etc. The instructor encouraged participants to use this procedure to estimate the interest they were paying on any loans they might have. This topic was not

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work-related for most participants, but is a useful concept to understand, participants seemed to appreciate the information and the practice added to participant's understanding of the calculator.

Performance Assessment Methods and Results:

- Working with the Calculator instructions was an important "extra" for the class, as it involved practicing reading skills, understanding and following directions, understanding the math concepts involved, problem-solving, and working from limited information.
- Participants found working with decimals easy, and enjoyed working with them on the calculator as it makes the task so much faster and easier.
- Percents did not come across to the participants as being quick and easy. The instructor tried to work with mental math of 10%, 25%, and $33\frac{1}{3}\%$ -- but because it was a new concept to them, it took more explanation and individual attention than anticipated. The participants needed to learn what to do on the calculator, and they did not find it easy. These were concepts that needed to be taught rather than just reinforced.
- The number of things participants could accomplish with just a cheap, simple calculator amazed everyone in the class.

Comments:

- The calculator being used did not do fractions, so the class discussed only the concept of how to do that task on a calculator. In the company offering this training, fractions are not used much, so the employees had little interest in working with them. The class spent very little time on them.
- The worksheet "*Compounding Things*" serves as a good introduction and discussion starter for the concept of compounded interest.
- The instructor used the calculator instructions to figure out how to get a right answer using the calculator. Because calculators tend to do similar things in subtly different ways, learning to read those instructions is critical to continuing success in using a wide variety of calculators. More importantly, the instructor wanted the participants to gain experience in referring to instructions detailing how a specific calculator works based on the design of that piece of equipment. If you are familiar with figuring out terse, incomplete directions, you can get what you need from them--especially if you already have some sense of what a calculator can do.
- Calculate! Problem Solving with Calculators has many other potentially useful worksheets to provide calculator practice with decimals and percents.
- This was a very conversational class. There was really very little lecture. More often, the class was participants talking among themselves and discovering together how things worked.
- The class did not do as much problem solving related to job tasks as the instructor would have liked. In the future, the instructor plans to include more problem solving.

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Activity VI: Understanding and Using a Calculator's Memory Functions

1. Worked with memory functions on the calculator
2. Looked for applications in the workplace where the use of memory functions would be appropriate.

Resources and Materials Needed:

- "Checking Out the Calculator's Memory" (p. 9) from Calculate! Problem Solving With Calculators by Terrence G. Coburn, Creative Publications, Mountain View, CA. 1988.
- "Adding Products With the Memory Keys" (p. 10) from Calculate!
- "Calculator Practice with Memory" Mary Mayfield, Instructor, Lane Community College, 1996.
- "Calculator Practice" Mary Mayfield, 1996.
- Instructions from the calculator's package
- Overhead projector with transparencies

Description of Instruction:

1. The class used the instructions accompanying the calculator to practice with the memory functions of the calculator. Participants first individually studied the calculator directions related to using the memory and tried to understand and use it. In pairs, they then discussed or explained to each other how they thought the memory worked. Finally the instructor led the a group explanation of the process, adding information as necessary.
2. "Checking Out the Calculator's Memory" and "Adding Products with the Memory Keys" from Calculate!, gave participants practice in using the calculator's memory keys. The participants and the instructor discussed the worksheets out loud as they worked through them. Participants interacted frequently between themselves, as well with the instructor. The calculator the participants used was different from the calculator described in the text (3 memory keys instead of 4, clear memory and delete memory were the same key). As a result, the class had to talk about how to use their calculators to perform the activities asked for in the text.
3. The instructor used an overhead of "Calculator Practice with Memory" with each participant having their own copy to work on individually. The class found it helpful to talk about using memory and then work with solving each problem together via the overhead. Participants worked in pairs to determine the keystrokes needed and then shared their results with the group.
4. The class used "Calculator Practice" as a review. Participants had to decide the most efficient way to solve the problem: straight computation, a constant, or the memory

Course Title: Using Calculators in the Workplace

function. The worksheet is a way to encourage participants to think about the best way to use the calculator.

Performance Assessment Methods and Results:

- The participants were successful at using the memory keys. They needed more practice with using memory, but time and lack of materials for practice prevented that. They understood how to use the memory function, but whether it was cemented enough to use on the job is not certain.

Comments:

- The resources available were not excellent. The concept of the "*Checking Out the Calculator's Memory*" was appropriate, but it was too complex to be comfortable and understandable for this group. It included some notation that was unfamiliar to the group. All of the worksheet examples used decimals, and for this group it would have been better to introduce the concept with whole numbers. It was too complicated for the purpose of just learning the memory key function.
- "*Adding Products with the Memory Keys*" was good. Although somewhat boring, it did show what you could do with memory keys.
- Calculate! Problem Solving with Calculators was a good resource. Otherwise, the instructor found very little in the way of commercial resources to call upon for this class. Given the time frame for the class, the instructor did not have time to develop them.
- The class was not as grounded in each of the concepts as the instructor would have liked because the time frame of the class didn't allow for enough practice.
- There needed to be more discussion about the benefits of using the memory key in a work setting.

Course Title: Using Calculators in the Workplace

Activity VII: Working with Large Numbers and a Calculator's Display

1. Participants learned how calculators deal with numbers too large to display on the calculator.
2. The class used the instructions accompanying the calculator to review how the participant's calculator handles large numbers.

Resources and Materials Needed:

- "Checking Out Large Products" (p. 12) from Calculate! Problem Solving With Calculators by Terrence G. Coburn, Creative Publications, Mountain View, CA. 1988.

Description of Instruction:

1. The instructor showed the class what the calculator does with numbers too large for the display. When the display of the answer is followed by a subscript E, it means the number is too large to show in the display area. The participants knew the calculator had reached its limit when they saw this error. They did not know the calculator would allow a user to complete one more step in a calculation after this message. The instructor explained that the answer for either of these calculations (the one showing the subscript E or the final calculation) would be whatever was shown in the display multiplied by 10^8 . (This, then, generated some discussion of powers of 10 and of moving the decimal point in a number to multiply by powers of 10.) The instructor also noted that these answers are only approximate as numbers had been dropped at the end because they do not fit the display.
2. The class used "Checking Out Large Products" for some practice with this function.

Performance Assessment Methods and Results:

- Many participants felt this process was very complicated and did not really grasp what was happening. The class needed more time and practice to really solidify what the calculator was doing before the participants would really grasp what this calculator capability can do for them.

Comments:

- The class said they did not work with large numbers very frequently and did not feel they needed this information. They went through it in a sketchy way to give them a sense of what the calculator is capable of doing. That instruction provided some background if they were to need this function in the future.

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Activity: VIII. General Introduction to Different Types of Calculators

1. The instructor explained and worked with the logic a calculator uses to help participants gain more information about the logic of the 4-function, 3 or 4-key memory calculator.
2. The class compared problems using a scientific calculator, and a 4-function calculator and checked the answers. Often the answers varied because the two calculators were using different logic to solve the problems.

Resources and Materials Needed:

- "Checking Out Your Calculator's Logic" (p. 11) from Calculate! Problem Solving With Calculators by Terrence G. Coburn, Creative Publications, Mountain View, CA. 1988
- A standard 4 function calculator
- A scientific calculator

Description of Instruction:

1. Participants who had extra time worked a little with the calculator's logic and the "Checking out your Calculator's Logic" worksheet.
2. The class discussed what you do if you pick up a calculator that is both unfamiliar and complicated. Participants benefit from knowing how to perform a few calculation tests with an unfamiliar calculator. That helps them understand how a calculator will process the calculations they feed it. They also learned basic differences between the simple type of calculator they used in class and a scientific calculator.

Comments:

- "Checking Out Your Calculator's Logic" worksheet was unclear and confusing for most participants, partly because it introduced in an offhand way the idea of order of operations that was unfamiliar to participants. This worksheet looks at calculator logic and provides a review of memory and the keystrokes needed to do certain operations before others. One way to do this might be:
 - 1) Have both basic and scientific calculators available
 - 2) Have participants try out each problem on each calculator
 - 3) Write down those results and compare them
 - 4) Key the problem into their basic calculator, using the keystrokes indicated on the right side of the worksheet
 - 5) Write this result and compare with the first results
 - 6) Discuss what is happening here and how to make use of it
- This was a very brief overview. If participants show interest, or if they use more than one type of calculator, more time could be spent on this kind of information.

Activity: IX. Concluding Activities
--

Resources and Materials Needed:

1. Complete grant paperwork.
2. Give "Calculator Post-test."
3. Provide feedback to participants, and get feedback from participants on class.
4. Complete Individualized Learning Plans (Part 1 was completed at the beginning of class).

Description of Instruction:

1. The class completed the paperwork for the grant.
2. The class took the "Calculator Post-test."
3. Instructor talked individually with each participant about their progress in the class and about how they felt about the class.
4. Each participant completed their Individualized Learning Plans.

Performance Assessment Methods and Results:

- Participants had the opportunity to review the progress between the pre and post test results.
- Participants also had a chance to provide input to the instructor on the quality of the class.

Course Summary

Background:

The Job Task Analysis report identified basic calculator operations as a training need. As a result, the company scheduled two calculator classes. Employees from a variety of departments comprised one class. The other class consisted of members from the shipping department. A member of the Site Advisory Team from the shipping department promoted the latter class. He proved instrumental in developing interest and organizing schedules for members of his department.

Expectations:

Company:

Employees in production, shipping, and the office use calculators on a regular basis in their daily work. Even with the use of calculators, employees make errors on production sheets. The course set out not only to cover calculator functions but to develop better mental estimations that would indicate whether the calculator answer was correct.

Learner:

Participants indicated interest in learning to do calculations more effectively, and in learning to use calculator memory and calculation constants. All participants used calculators on a daily basis.

Instructor:

In surveying curriculum for this course, the instructor discovered a limited amount of published material available for calculator instruction. Consequently, she used the directions for the calculator as a starting point for the course and fleshed out the curriculum by drawing on the experience and skill level of the participants.

Evaluations:

Company:

The calculator class as a stand-alone training did not have large numbers of participants. However, because using the calculator efficiently is a priority at the worksite, the longer scheduled math trainings that most production workers attended incorporated use of the calculator.

Course Title: Using Calculators in the Workplace

Learner:

Participants indicated that they met the top three objectives that they set at the beginning of training--understanding memory, calculation constants, and practicing calculator directions.

Instructor:

The instructor felt that the course required more than five sessions so that students could learn the functions of the calculator and have sufficient time to practice work-related uses. In planning the class again, she would add more time for practicing specific work functions.

Handouts

Calculator Pretest

Name _____

Date _____

1. Name at least 2 advantages of using as few keystrokes as possible to do your calculations.

2. You need to find: $37 + 49$. You have entered 3 7 + 9. What do you do to erase just the 9 so you can enter the 49 correctly?

3. You realize that you have entered the wrong numbers for your whole calculation. What do you do to clear that calculation so you can start over--without erasing anything you have put into memory?

4. What happens if you press the R&CM key twice? _____
5. After a series of calculations, your display shows 2.819361E, what does this mean?

6. Use the calculator to change $7/16$ to a decimal. Write your keystrokes on the spaces below.

7. Use your calculator to do the following series of calculations. Write your keystrokes on the spaces below. $295 \times 27 \times 18 + 122 - 79$

8. The perfect weight of a length of pipe is 54 lb. The pipe is 1.2% overweight. Use your calculator to find the actual weight of the pipe. Write your keystrokes on the spaces below.

9. What calculator keys would you press to get the following displays? Try to use as few keys as possible.

_____ 3 _____ 9 _____ 27 _____ 81 _____ 243

10. You have crates of pipe weighing 2887 lb., 2875 lb., and 2883 lb. Each crate weighs 44 lb. Use the spaces below to write the calculator keystrokes you would use to most quickly and efficiently find the total weight of the pipe you are about to ship. (Put 1 key stroke per blank. You may or may not use all the blanks.)

Learning Log for Calculator Training

Date	Concept Learned	General Practice	Application: Self	Application: Job	Application: Family/Home	Comments/Questions How did it work?

223

224

Checking Out the Addition Constant

Some calculators store one of the numbers in an addition equation as a constant.

1. Mark the keystrokes that match what your calculator shows.

☐ 2 ☐ + 3 ☐ = ☐ = ☐ = ☐ =

If your calculator does this, it uses the second number as the constant.

☐ 2 ☐ + 3 ☐ = ☐ = ☐ = ☐ =

If your calculator does this, it does not have an addition constant feature.

2. If you have the addition constant feature, you can use it for skip counting. Try these. Watch your display. Record the last sum.

Count on by 1s: ☐ 1 ☐ + ☐ = ☐ = ☐ = ☐ = ☐ =

Count on by 3s: ☐ 3 ☐ + ☐ = ☐ = ☐ = ☐ = ☐ =

Count on by 200s: ☐ 200 ☐ + ☐ = ☐ = ☐ = ☐ = ☐ =

Count on by 2020s: ☐ 2020 ☐ + ☐ = ☐ = ☐ = ☐ = ☐ =

3. You can also use the addition constant feature to add one number to lots of different numbers. You will only need to enter the constant once.

Press ☐ 17 ☐ + 17 ☐ = . Now 17 is in your calculator as the constant. Do not press ☐ or you will clear the constant.

Now press 9 ☐ = Press 25 ☐ = Press 64 ☐ =

Try adding 19 to these numbers. Press ☐ 19 ☐ + 19 ☐ = to enter the constant.

16 ☐ =

23 ☐ =

46 ☐ =

83 ☐ =

76 ☐ =

919 ☐ =

Name _____

Practice with the Constant Feature of Your Calculator

Use the constant feature of your calculator to solve the following problems. Write your answer inside the parentheses and write your keystrokes on the line.

1. To change a fraction to a decimal, you divide the top number by the bottom number. Change the following fractions to decimals:

$7/8$, $3/8$, $5/8$ () _____

2. You have crates of pipe weighing 947 lb., 896 lb., 922 lb. and 912 lb. The board weight of each crate is 23 lb. List below the weight of pipe in each crate.

947 lb. (lb.), 896 lb. (lb.), 922 lb. (lb.), 912 lb. (lb.)

3. You have crates of pipe weighing 1211 lb., 1202 lb., and 1217 lb. The board weight of each crate is 37 lb. Figure the total weight of the pipe.

(lb.) _____

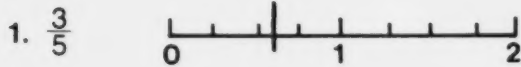
4. To find the average of a group of numbers, you first find the total, then divide by the amount of numbers you totaled. Find the average footage of pipe produced per shift this week if footages produced were 1200, 1600, 1200, 1400, 1600, 1600, 1400, 1200, 1600, 1200, 1000, 1200, 1400, 1200.

(ft.) _____

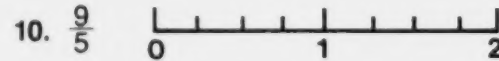
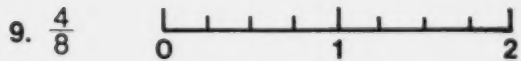
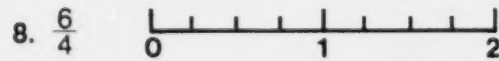
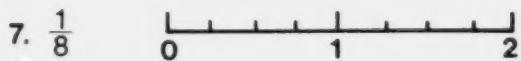
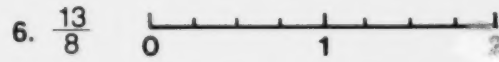
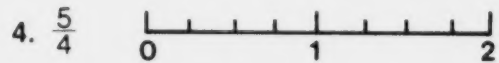
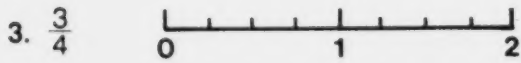
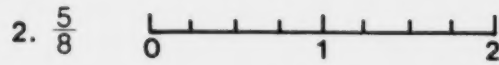
5. You are in charge of ordering supplies to set up 5 new work stations. For each work station, you will need a stapler (\$12.95), staples (\$3.19), a tape dispenser (\$5.95), a log book (\$6.75), a computer (\$1735.00), a box of computer disks (\$14.95), a package of pencils (\$.89), a package of pens (\$1.29), and paper clips (\$2.25). Find the subtotal for each of these items.

Changing Fractions to Decimals

Divide to find the decimal equal to each fraction. Write the keystrokes. Mark the number line to show where each fraction and its decimal equivalent belong.



$3 \div 5$



Percent Problems

Use your calculator to solve the problems. Write the keystrokes.

1. The light bulb company made 2.5 million bulbs last year. 0.6 percent of the bulbs were defective. How many of the bulbs were defective? _____

☐ C

2. The dairy has 12,860 pounds of milk in its containers. 4.15% of the milk is butterfat. How many pounds of butterfat are in the containers? _____

☐ C

3. 120 children are in camp. 60 percent can swim. How many can not swim? _____

☐ C

4. Matt earns 13% on the sale of newspapers. Last week he sold \$426 worth of newspapers. How much did he earn? _____

☐ C

5. Suppose your brother's estimated expenses for college are:

Room and board	3750
Tuition	4160
Books	890
Other	880

What percent is tuition of the total expenses? _____

☐ C

6. A new pair of cotton slacks will probably shrink 2.5%. The legs of the new slacks are 32 inches long. How long will they be after washing? _____

☐ C

7. Between 1970 and 1980, the population of Michigan increased from 8,882,083 to 9,275,000. What percent increase is this? _____

☐ C

8. The Jalopy Car Company recalled 12.5% of the cars it built in one year. In all, it recalled 154,320 defective cars. How many cars did the company build? _____

☐ C

9. A camera listed at \$195.99 was sold at a discount of 33.33%. What was the selling price? _____

☐ C

10. A book salesman earns a base salary of \$32,850 per year and an 11.75% commission on all sales over \$250,000. What was the total salary last year if \$462,500 worth of books were sold? _____

☐ C

Use the constant factor feature of the calculator to calculate the balance on \$500.00 earning 6% interest compounded each year for five years.

- | | |
|-----------|-----------|
| $\square$ | 669.11278 |
|-----------|-----------|

- Compounding interest adds the interest to the principal.
- Multiplying 500 by 1.06 is the same as adding 6% of 500 to 500.
- Each time the $\square$ key is pressed, the calculator shows the balance after that year.

The balance after five years is \$669.11.

Use your calculator to solve the problems.

1. You place \$200 in a bank where the money will earn compounded interest of 8.15% each year. How much money will you have in 12 years?

C 1.0815 x 200

2. If inflation is increasing prices at a rate of 3.5% per year, how much will a 65 cent can of soda cost in 7 years? _____

C 1035 x 65 = = = = =

3. You put 5 dollars in a bank that is paying 13% interest compounded annually. How many years will it take before you have \$10 or more? Count and write the number of times you press ☐. _____

4. Mr. Herman's salary is now \$26,500 per year. He expects that his salary will rise at a rate of 3% each year. How many years will it take before his salary is \$40,000 or more?

C

Checking Out the Calculator's Memory

Four keys control this calculator's memory.

CM clears the memory. Zero is in the memory when it is clear.

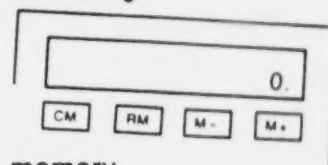
RM reads the memory. This is a display of the total stored in the memory.

M+ adds the number shown in the display to the memory.

M- subtracts the number shown in the display from the memory.

The letter M appears somewhere in the display when the memory has a number other than zero stored.

Try these examples. Remember to clear the memory **CM** before each new equation.



- | | Keystrokes | |
|---|--|---------------------|
| 1. $(6.2 \times 2) + (7.1 \times 3)$ | CM C 6.2 x 2 M+ 7.1 x 3 M+ RM | 33.7 ^M |
| 2. $\frac{1.2}{4} + \frac{2.5}{5}$ | CM C 1.2 ÷ 4 M+ 2.5 ÷ 5 M+ RM | 0.8 ^M |
| 3. $0.5 + 0.5 + 0.5 + 0.5 - 0.7$ | CM C .5 M+ M+ M+ M+ .7 M- RM | 1.3 ^M |
| 4. $0.5 - (0.07 \times 0.07)$ | CM C .5 M+ .07 x .07 M- RM | 0.4951 ^M |
| 5. $(1.5 \times 80 - \frac{2.4}{.06}) \times 1.1$ | CM C 1.5 x 80 M+ 2.4 ÷ .06 M- RM x 1.1
= | 88. ^M |
| 6. $(0.15 \times 0.11) - (0.14 \times 0.12)$ | CM C .15 x .11 M+ .14 x .12 M- RM | 0.0003 ^M |
| You write the correct keystrokes for these. | | |
| 7. $(1.2 \times 3) + (1.5 \div 3)$ | CM C _____ | 4.1 ^M |
| 8. $\frac{100}{0.4} - \frac{25}{0.5}$ | CM C _____ | 200. ^M |
| 9. $21.3 + 21.3 + 21.3 + 21.3$
$- 1.7 - 1.7$ | CM C _____ | 81.8 ^M |
| 10. $8.7 - (2.3 \times 2.3)$ | CM C _____ | 3.41 ^M |
| 11. $(1.2 \times 5 - \frac{4.5}{0.9}) \times 4.5$ | CM C _____ | 4.5 ^M |
| 12. $(150 \times .33) - (275 \times .44)$ | CM C _____ | 71.5 ^M |

Adding Products with the Memory Keys

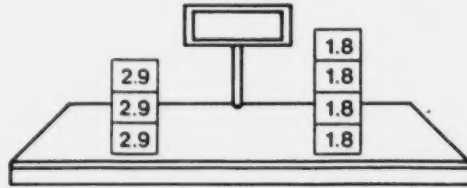
Use the **M+** **RM** **CM** keys to add two products.

For example:

Find the total weight.

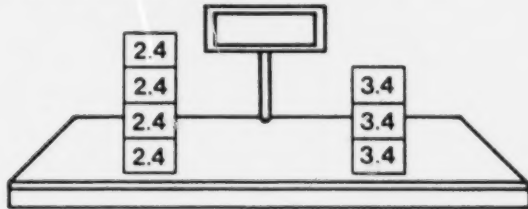
Add 3×2.9 and 4×1.8 .

CM **C** 3 **x** 2.9 **M+** 4 **x** 1.8 **M+** **RM** 15.9



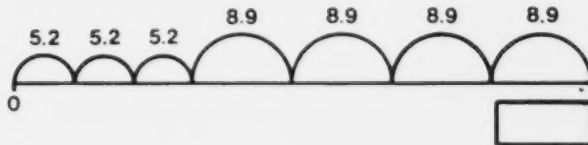
Use the memory keys to help you add these products. Write the keystrokes. Be sure to clear the memory before each new problem. The M symbol should disappear.

1. Find the total weight.



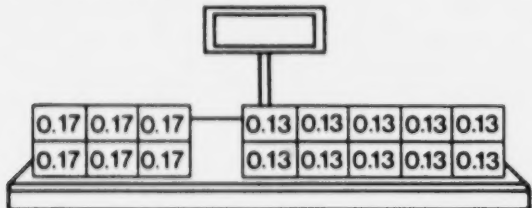
CM **C**

2. Find the landing spot.



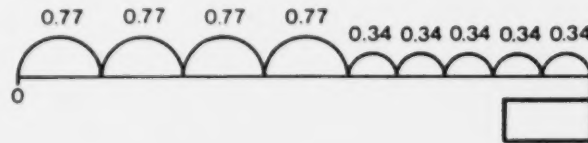
CM **C**

3. Find the total weight.



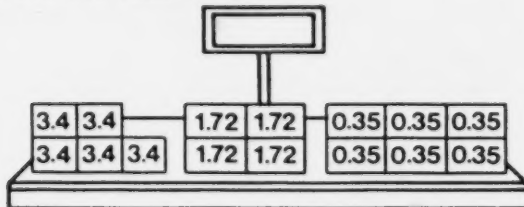
CM **C**

4. Find the landing spot.



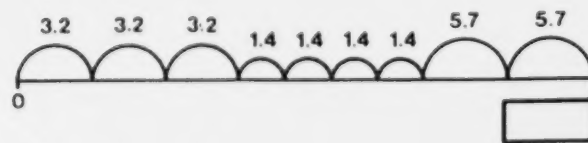
CM **C**

5. Use the **M+** key three times.
Find the total weight.



CM **C**

6. Use the **M+** key three times.
Find the landing spot.



CM **C**

Calculator Practice with Memory (and more)

You are hosting a party. You need to buy 5 half-gallons of ice cream at \$2.59 each, 2 cakes at \$7.75 each, 8 six-packs of beer at \$5.29 each, 6 giant bags of chips at \$3.19 each, and 6 cartons of dip at \$3.55 each. What will these supplies cost you?

You loaded a container of nested pipe that has 12 pieces of 24" pipe, 12 pieces of 18" pipe, 9 pieces of 15" pipe, 15 pieces of 12" pipe and 15 pieces of 6" pipe. All pipe is 20' long. Figure the number of feet of each size of pipe in this load and the total feet of pipe in the load. Use the calculator for all steps.

You have crates of pipe weighting 1211 lb., 1202 lb., and 1217 lb. The board weight of each crate is 37 lb. Figure the total weight of the pipe.

Name _____

Calculator Practice

You now know several different ways to approach computations with your calculator. For each of the following problems, decide whether to do a) straight computation, b) computation using a constant or c) computation using the memory; then solve each problem, using the method you chose. Write the answer to the problem in the parentheses and write the keystrokes you used on the line. (Note: For practice, please try to choose what you think would be the quickest, most efficient method. Also, hopefully you'll choose different methods for different problems.)

1. To change a fraction to a decimal, you divide the top number by the bottom number. Change the following fractions to decimals:

5/16, 9/16, 11/16()

2. You have crates of pipe weighing 974 lb., 869 lb., 922 lb. and 921 lb. The board weight of each crate is 26 lb. List below the weight of pipe in each crate.

974 lb. (lb.), 869 lb. (lb.), 922 lb. (lb.), 921 lb. (lb.)

3. You are shipping an order of pipe which includes 24" pipe weighing 7,050.4 lb., 18" pipe weighing 4,006.1 lb., 15" pipe weighing 2,684.7 lb., 10" pipe weighing 1,641.6 lb. and 3" pipe weighing 136.8 lb. What is the total weight of the pipe shipped in this order?

(lb.)

4. You are driving toward your hunting destination. The first 1.5 hours you are traveling the freeway and average 67 mph. Then next 2.5 hours, you are on a paved secondary road that winds around the mountain as it climbs, so you average only about 45 mph. The final hour you travel about 25 mph on a gravel forest road. Approximately how many miles did you travel?

(mph) _____

5. You are doing an inventory of 8" and 10" pipe. There are 7 crates of 8" pipe with 200' per crate and 14 crates of 8" pipe with 300' per crate plus an additional 11 crates of 10" pipe with 160' per crate. Find the stored footage of each size of pipe and the total footage for both sizes.

8" (ft.), 10" (ft.), TOTAL (ft.)

5. You are in charge of ordering supplies to set up 5 new work stations. For each work station, you will need a stapler (\$12.95), staples (\$3.19), a tape dispenser (\$5.95), a log book (\$6.75), a computer (\$1735.00), a box of computer disks (\$14.95), a package of pencils (\$.89), a package of pens (\$1.29), and paper clips (\$2.25). Find the subtotal for each of these items; then the total for the whole order.

Staplers(_____), Staples(_____), Tape dispensers(_____), Log books(_____)

Computers(_____), Disks(_____), Pencils(_____), Pens(_____)

Paper clips(_____), TOTAL (_____)

Calculator Post-test

Name _____

Date _____

For the first 5 questions, you may look at your calculator for reference, but please don't use it to experiment with possibilities to find the answer.

- 1 Name at least 2 advantages of using as few keystrokes as possible to do your calculations.

- 2 You need to find: $73 + 94$. You have entered 7 3 + 4. What do you do to erase just the 4 so you can enter the 94 correctly?

- 3 You realize that you have entered the wrong numbers for your whole calculation. What do you do to clear that calculation so you can start over--without erasing anything you have put into memory?

- 4 What happens if you press the R&CM key twice? _____

- 5 After a series of calculations, your display shows $2.819361E$, what does this mean?

Use your calculator to solve these final 5 questions. If there seems to be more than one way to approach solving it, try to choose a way that uses as few keystrokes as possible.

- 6 Use the calculator to change $13/16$ to a decimal. Write your answer inside the parentheses and write your keystrokes on the line after the parentheses.

() _____

7. Use your calculator to do the following series of calculations. Write your answer inside the parentheses and write your keystrokes on the line. $2600 \times 15 - 660 + 1820$

() _____

8. The perfect weight of a length of pipe is 128.5 lb. The pipe is 1.05% overweight. Use your calculator to find the actual weight of the pipe. Write your answer in the parentheses and write your keystrokes on the line.

() _____

9. Write a possible real world math problem that could efficiently be solved using one of the constant functions (addition, subtraction, multiplication, or division constant) on your calculator. (Be clear and specific--so I can understand the problem. But don't worry about mistakes in grammar, spelling, etc.) Then solve the problem, writing the answer in the parentheses and the keystrokes on the line.

() _____

10. A load of pipe has 9 pieces weighing 720 lb. each, 4 pieces weighing 256 lb. each, 3 pieces weighing 130 lb. each, and 8 pieces weighing 161 lb. each. Use the memory function of your calculator to solve this problem efficiently. Write your answer in the parentheses and your keystrokes on the line.

() _____

BIBLIOGRAPHY

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"Any Which Way"(p.66), Calculate!

"Calculator Practice" Mary Mayfield, Instructor, Lane Community College, 1996.

"Calculator Practice with Memory" Mary Mayfield, Instructor, Lane Community College, 1996.

"Changing Fractions to Decimals"(p. 53), Calculate!

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"Checking out the Addition Constant " (p. 3), Calculate!

"Checking Out the Calculator's Memory" (p. 9) Calculate!

"Checking Out the Division Constant" (p. 8), Calculate!

"Checking Out the Multiplication Constant"(p. 7), Calculate!

"Checking Out the Subtraction Constant" (p. 4), Calculate!

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"Percent of Increase or Decrease"(p.75), Calculate!

"Percent Problems" (p.81), Calculate!

"Practice With the Constant Feature of Your Calculator" Mary Mayfield, Instructor, Lane Community College, 1996.

Calculator Power: The Complete Handbook by Robert Mitchell, Contemporary Books, Chicago, IL, 1991 pp. 13, 15, and 17.

Workplace Training Project

Course Title: Algebra / Geometry

Course Description:

The primary focus of this class was on three areas: ratio and proportions, geometry, and algebra, with emphasis throughout on thinking skills and problem-solving. There was also time to review number concepts, estimation, basic operations with fractions and decimals, and percents. It was also important that instruction be responsive to skills and needs of individual participants. The math skills taught had to be as relevant as possible to the work participants did, using actual workplace examples whenever possible.

Review of basic concepts and operations took longer than anticipated. Virtually all participants wanted significantly more instruction and practice with percents than planned. Participants felt a lack in the area of percentages, and they are used quite a bit on the job. As a result, the worksite and the project staff extended the class to allow more time to be spent on algebra and geometry.

Student Outcome Objectives

Participants will demonstrate the ability to:

1. Work with and talk about examples and activities that help them visualize, integrate and understand basic number concepts, patterns and relationships.
2. Develop some understanding of the importance of estimation and some skill in using estimation.
3. Review decimal and fraction concepts and operations, practicing these skills in ways used on the job or in everyday life.
4. Work with proportions (both direct and indirect variation) and apply both in ways that are useful on the job.
5. Work with percents as a ratio (compared to 100), develop some understanding of the relationship between fractions, decimals and percents and practice using proportions as a way to solve percentage problems.
6. Practice solving practical percentage problems (using proportions or another way that they feel comfortable with).
7. Work with exponents and with square roots, work with scientific notation, and work with the Pythagorean theorem (using this concept to solve practical problems).

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8. Work with some properties of angles, lines and triangles.
9. Work with the concepts, formulas for and applications of perimeter, area and volume of standard 2- and 3-dimensional figures.
10. Work with algebraic concepts such as writing and solving equations from word problems, graphing equations, and writing equations based on collected data.

Student Population:

This was a group of 7 employees of a manufacturing company. Most worked in the maintenance department and most had math and problem-solving skills above the level of those of the average company employee.

The instructor adapted part of this curriculum to teach a more basic class in pre-algebra. That class also had about 7 participants (it fluctuated some). They were mostly production workers. Some had just completed the Basic Math Training and most had math and problem-solving skills and experience at a somewhat lower level than the original group.

Class Environment:

Algebra/geometry classes ran 20-24 class sessions, Pre-algebra ran about 20 class sessions, for approximately 1.25 hours per class session once or twice a week. The class took place in the plant conference room. It was a comfortable and appropriate room for classes, especially for classes of a dozen or fewer participants. There were chairs with padded arms and plenty of table space. When necessary, the instructor could rearrange the space. There was a small white board and an overhead projector.

Participants all knew each other and were pretty open about asking questions and participating, although most were not very secure in their math skills. One participant in particular did not like doing puzzle-type problems and all wanted clear relationships to practical use.

Instructional Techniques: ✓

☐ Multi-media	☒ Small group	☒ Cooperative learning
☒ Lecture	☐ Computer-aided	☒ Assignments
☒ Individualized instruction	☐ Simulation	☐ Peer tutoring
☐ Role-playing	☐ Project	☒ In-class practice
Other:		

Algebra / Geometry

Syllabus of Course Activities:

I. Preview

- Introductory information about the class
- Students fill out Preview form, with information about background and expectations
- Grant paperwork
- Introductory assignment

II. Review and Background

- Number concepts and patterns
- Estimation
- Decimals (comparing decimals)
- Fractions

III. Ratio and Proportion

- Understanding ratio
- Direct variation
- Inverse variation

IV. Percents

- Visualizing and understanding the concept of percent
- Relationships between fractions, decimals, and percent
- Using a proportion to solve percent problems
- Other ways of solving percent problems
- Percent applications on the job and in everyday life

V. Exponents, Roots and Scientific Notation

VI. Angles, Lines, Triangles including the Pythagorean Theorem

VII. Computing with Measurements

VIII. Perimeter, Area, Volume

- Perimeter as distance around straight-sided figures
- Area of triangles, rectangles, squares, trapezoids, parallelograms
- Circumference and area of circles
- Surface area

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Perimeter, Area, Volume (continued)

- Volume of cubes, rectangular solids, cones, cylinders, spheres, pyramids
- Application of area and volume formulas

IX. Basic Algebra Concepts

- Signed numbers
- Order of operations
- Algebraic expressions and formulas

X. Working with Equations

- Solve linear equations in one variable
- Use linear equations to solve word/work problems

XI. Working with Graphs

- Interpret charts and graphs
- Graph linear equations in two variables
- Determine and use slope and intercepts
- Use graphs and equations to interpret or make predictions from collected data

XII. Concluding Procedures

Activity I: Preview

1. Introductions, general overview of class
2. Participants filled out "Algebra/Geometry Preview" form
3. Discussed with participants their expectations for the class
4. Filled out Grant paperwork
5. Used pretest to assess skills

Resources and Materials Needed:

- "Algebra/Geometry Preview" Mary Mayfield, Instructor, Lane Community College, 1996. (This was similar to the "Math Training Preview," which is included with the Basic Math curriculum.)
- # "Algebra/Geometry Outline" Mary Mayfield, 1997.
- # "Algebra/Geometry Assignment" (located on bottom of "Algebra/Geometry Outline" page) Mary Mayfield, 1997.
- Grant paperwork, including Individual Learning Plan with class objectives for participants to review
- Pretest
- "Characteristics of Today's and Tomorrow's Workplace" Excerpt from What Work Requires of Schools: A SCANS Report for America 2000, U.S. Department of Labor. Secretary's Commission on Achieving Necessary Skills, Washington DC, 20210, 1990.
- # # "Math Anxiety Bill of Rights" Mary Mayfield, 1997. # # (Copy of this included with Basic Math curriculum)
- # # "Ground Rules" Mary Mayfield, 1997. # # (Copy of this included with Basic Math curriculum)

- A copy of the marked handout is in the **Handout** section of this curriculum.

- A copy of the marked handout is in the **Handout** section of a different curriculum.

Description of Instruction:

Before the actual class started, the instructor held a "preview" session. This session gave participants a general overview of the class and asked for input from potential participants. The instructor came with a list of possible topics to be covered and added to or modified this list in discussion with those attending this session. The participants who were planning to attend the class filled out "Algebra/Geometry Preview" and talked about their expectations and their background, as an aid to the instructor in planning this class. Another reason for the preview

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session was to be sure that participant interest and skills were reasonably well matched with the planned content of the class.

Then, the first regular class session got the class off to its official start.

1. Participants filled out the grant paperwork
2. Participants filled out the Individual Learning Plan with the class objectives specified on it.
3. The instructor answered questions about class content and structure based on the "*Algebra/Geometry Outline*." She also handed out and discussed "*The Math Anxiety Bill of Rights*," "*Ground Rules*," and "*Characteristics of Today's and Tomorrow's Workplace*."
4. Instructor wanted to know more about participants' actual jobs to help her customize the class to meet their needs. To gather some of this information, she gave the participants "*Algebra / Geometry Assignment*" (which is on the same page as the outline). Three participants brought their completed assignment after the first week. (By the 3rd class meeting they had all brought it.)
5. Participants took the pretest.

Comments:

- The instructor emphasized this class would be a joint venture, that participant input would be valuable, and that she would customize the class to answer their questions, meet their needs, etc.
- Participants were not really into "school mode." They expressed willingness to do out-of-class assignments and most participants usually did at least some of the homework. Still, because they already had busy work and personal lives, homework was not the first priority.

Activity II: Review and Background

1. Number concepts and patterns
2. Estimation activities
3. Review of decimals
4. Review of fractions

Resources and Materials Needed:

- ## "Open Calculation" Mary Mayfield, Instructor, Lane Community College, 1997. ## (Copy of this included with Basic Math curriculum.)
- ## "Number Patterns, Problem-Solving, and Review" Mary Mayfield, 1996. ## (Copy of this included with the Basic Math curriculum.)
- ## "Finding Averages," Mary Mayfield, 1995. ## (Copy of this included with the Basic Math curriculum.)
- Base 10 blocks or other manipulatives as needed for demonstration
- "Mental Imagery," Visual Mathematics, The Math Learning Center, Salem, OR, 1991.
- Contemporary's Fractions, booklet of Math Exercises. Robert P. Mitchell, published by Contemporary Books, Inc., Chicago, Illinois. Copyright 1993.
- Contemporary's Decimals, Math Exercises. Robert P. Mitchell. Published by Contemporary Books, Inc., Chicago, Illinois Copyright 1993

Description of Instruction:

NOTE: For the **Review and Background unit**, the instructor used selected worksheets and activities that are included in the **Basic Math curriculum**. Some possible selections are noted here; further information on activities or materials related to this unit can be found in **Basic Math: Activities II through V**. Any additional materials or activities used specifically for this unit will be discussed in full here.

1. The instructor used worksheets such as "*Number Patterns, Problem-Solving, and Review*," "*Open Calculation*," and "*Finding Averages*" to review number concepts and computations in ways that would encourage participants to:
 - look for patterns
 - to problem-solve
 - to ask questions
 - to talk about what they know and how they think
 - understand the "inner workings" of number-related processes rather than just be able to do a rote process (multiply, find an average).

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2. "Mental Imagery" involved the instructor asking participants a number of questions such as "Where is the red light on a stop light?" or "On which side of the sink, as you face it, is the hot water tap?" or "Which sock do you put on first?"
3. The instructor scattered estimation activities throughout the course. For example, the instructor might ask participants to estimate how much chocolate the average person would eat in a year. The idea was to try to get participants to go through all the steps necessary to come up with a reasonable estimate.
 - Does "chocolate" mean just chocolate as in chocolate candies--or does it include chocolate milk, chocolate chip cookies, mocha drinks, etc.?
 - How many people eat chocolate?
 - How much chocolate would one person eat in a day?
 - How much chocolate do I eat? Is that kind of the average? Do most people eat more or less than that?
 - How much would that be in a week? in a month? in a year?
4. Review of decimals was mostly a standard review of computation and comparison skills, using a selection of materials from the Basic Math curriculum.
5. Review of fractions was very brief. The instructor found that the participants really didn't use fractions much on the job. They did have to do some fraction/decimal conversions, mostly with eighths, sometimes with other fractions. The instructor presented a brief overview of fraction concepts and answered participant questions. Most participants remembered at least some fraction computation skills and some wanted some clarification of what they remembered. They practiced these skills with a few fraction word problems. Then the class discussed and practiced fraction/decimal conversion using pencil and paper, a basic 4-function calculator, and a scientific calculator with fraction keys.

Comments:

- Presenting "Number Patterns, Problem-Solving, and Review" as an in-class worksheet was a disaster. The instructor misjudged the level of the participants' readiness for this sort of math reasoning activity. The pretest and her other interactions with participants indicated that the participants seemed pretty comfortable and competent with basic computational math skills. She thought that a puzzle-type, thinking-skills math activity would broaden the way the participants thought about math. She expected the activity to arouse interest, create some early success in problem-solving and remind participants of some number concept relationships. Unfortunately, a 2-page worksheet of this type of problem just seemed overwhelming to the participants. Participants experienced the problems as "hard," not as interesting or challenging. They didn't see how to get started on them or understand the point of the activity. One participant was particularly vocal about disliking this kind of activity. This made it especially difficult for the instructor to build rapport. She wanted to use this activity to have fun and look at number concepts and number relationships. This experience prompted the instructor to use the activity in shorter, more directed ways--as presented in the Basic Math curriculum (Activity II:

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Whole Number Concepts, Number Relationships, and Computation). This approach worked much better.

- In the beginning, the class did fewer hands-on activities than the instructor would have liked, because some participants had real reservations about such activities (considering them childish or "fluff"). "*Mental Imagery*" was usually used as a warm-up activity, to initiate a discussion of the role visualization and kinesthetics play in our understanding, memory, and learning. The class discussed how doing hands-on activities as part of learning helps the learning "stick." Mental imagery works as a warm-up at the very beginning of the course. The instructor also uses it when participants begin working with manipulatives, or when some resistance to work with manipulatives surfaces.

Another technique the instructor used to allow participants to become more comfortable with using manipulatives was to begin by using them mostly herself for illustration/demonstration. Gradually, she asked a participant, a group of participants, or the whole class to use them for some purpose.

- The type of estimating activity discussed above was a good exercise in helping participants think logically to come up with a reasonable answer. Initially, however, many participants equated "estimate" with "guess." They did not think of estimating as a reasoned approach to a logical answer nor did they understand the importance of estimation in math or in their work. They, therefore, needed practice to understand and apply a thoughtful, logical process to make an estimate.

In addition, the company said that many mistakes employees made occurred because employees didn't recognize whether their answers were reasonable or unreasonable. The company specifically requested that math classes include estimation and reasoning skills.

- Participants understood comparison and computation with decimals quite well and moved through that section quickly. In the future, the instructor would use more activities such "*Hit the Target*" and "*Decimal Estimation*," (See Basic Math curriculum, Activity IV) to give participants more practice with decimal concepts and develop their "sense" of decimals.
- The class spent 3 or 4 sessions reviewing the math background concepts discussed above.

Activity III: Ratio and Proportion

1. Understanding Ratios
2. Direct Variation
3. Inverse Variation

Resources and Materials Needed:

- # # "Problems Stated as Ratios", GED Priority Math, "Ratio and Proportion" section. Mary Ann Schmidt, Massachusetts Department of Education, 1989. # # (Copy of this with the Basic Math curriculum.)
- # # "General Multiplication/Division Word Problems", GED Priority Math, "Ratio and Proportion" section # # (Copy of this with the Basic Math curriculum.)
- # # "Similar Shapes", GED Priority Math, "Ratio and Proportion" section # # (Copy of this with the Basic Math curriculum.)
- blocks or other small manipulatives
- # # "Using Proportions: Estimation of Fish Population" # # (Copy of this with the Basic math curriculum.) Copied from old discarded elementary math books -- copyrighted between 1975 to 1982.
- # # "Using Proportions: Bicycle Gears" # # (Copy of this with the Basic math curriculum.) Copied from old discarded elementary math books -- copyrighted between 1975 to 1982.
- # # "Using Proportions: Model Airplanes" # # (Copy of this with the Basic math curriculum.) Copied from old discarded elementary math books -- copyrighted between 1975 to 1982.
- # # "Practice with Ratios and Proportions" Mary Mayfield, Instructor, Lane Community College, 1996. # # (Copy with the Basic Math curriculum.)
- # # "Direct Variation" Mary Mayfield, 1996. # # (Copy with the Basic Math curriculum.)
- # "Inverse Variation" Mary Mayfield, 1996.
- # "Review of Direct and Inverse Variation" Mary Mayfield, 1996.

Description of Instruction:

NOTE: Materials and activities for the first half of this unit (introducing ratios and proportions) are selections from the **Basic Math curriculum (Activity VI: Working with Ratios and Proportions)**. For additional resources or for additional information about how some of the resources listed here might be used, please refer to that curriculum.

The class began a study of inverse variation when a participant asked a question that revealed there were many applications of this concept on the job. That material is new in this section.

1. The instructor gave a brief presentation of ratios using visible quantities (for example, number of people wearing T-shirts to total number of people -- or -- number of people wearing T-shirts to number wearing baseball caps). She gave participants "*Problems Stated as Ratios*," both the "Teacher Tips" page and the worksheet. She gave participants some time to look through the "Teacher Tips" examples. Then, she went through some of it with them, answering their questions and noting important concepts such as the directionality of ratios (the ratio of men to women means something different than the ratio of women to men) or importance of simplifying ratios.

Participants then worked on some of the problems from the worksheet, with the instructor monitoring and answering questions as they came up. Whatever problems weren't finished on the worksheet became homework.

2. As a warm-up activity to introduce proportions, the instructor wrote some analogies on the white board, such as "wet is to dry as swamp is to _____" or "wheat is to flour as _____ is to juice." These prompted some discussion that the instructor then led toward examples of equal ratios in the room. She also used some manipulatives (blocks, etc.) to show equal ratios. She wrote some of these equal ratios on the board or overhead projector as proportions. She further demonstrated that the cross-products of proportions will be equal. Next, participants had to explain, through discussion, how they might find a missing number in a proportion. During this discussion she gave participants "*General Multiplication / Division Word Problems*," both the "Teacher Tips" sheet and the worksheet. The class discussed the process used to solve the problem examples, then practiced setting up a few problems.
3. Participants then worked on "*Practice with Ratios and Proportions*" to practice solving problems using proportions and to begin to experience the variety of situations in which using a proportion might be appropriate. Participants worked some problems with the instructor monitoring and answering questions; then they did the rest as homework.
4. As a warm-up activity and review of proportions, the instructor had the group divide into pairs and had each pair study one of the following: "*Using Proportions: Estimation of Fish Population*," "*Using Proportions: Bicycle Gears*," "*Using Proportions: Model Airplanes*," "*Similar Shapes*." Participants were to read the materials, and do enough exercises to be sure they understood it. Part of the practice was to explain a problem to each other, then present what they learned to the rest of the class.
5. A participant brought up a need to use proportions that are inversely proportional because these show up in the workplace. (Two quantities vary inversely when an increase in one produces a decrease in the other--for instance: as your rate of speed increases, the time to cover a certain distance decreases -- or -- as electrical resistance increases, the amount of power goes down.) Most of the participants did not understand how to approach those problems.

The class first did the problems on the "*Direct Variation*" worksheet. This provided further practice with practical applications of direct variation. The instructor then introduced indirect variation with a definition and then discussion of quantities participants could think of that vary inversely. The worksheet "*Indirect Variation*" provided practice to gain skill in setting up inverse proportions. They especially had trouble determining which situations involved inverse variation and which involved direct variation.

6. The instructor suggested to participants that whenever they had a situation that seemed proportional, to first take a moment to decide whether the quantities varied directly (when one increases, the other increases) or inversely (when one increases, the other decreases). She also suggested that to simplify setting up inverse proportions, the 2 given quantities that are related to each other be written diagonally from each other (that is, one in the numerator of one ratio, the other in the denominator of the other ratio), then the third given quantity can be put in either of the remaining place.

Example: If 8 people do a job in 9 days, how long do 24 people take to do the same job? In this example, 8 people is related to 9 days, so these quantities could be written in as

$$\frac{8}{?} = \frac{?}{9} \text{ or } \frac{?}{8} = \frac{9}{?}, \text{ etc.}$$

The 24 can then be written in one of the remaining places, as $\frac{8}{24} = \frac{?}{9}$ or $\frac{8}{?} = \frac{24}{9}$, etc.

7. Participants were given "*Review of Direct and Inverse Variation*" for further practice in distinguishing between and working with direct and indirect variation.

Performance Assessment Methods and Results:

- Participants really liked working with proportions because they could see the application for them in their work immediately. One participant said, "Oh, I can use this. This works!" Production people at this plant used this because when they speed up or slow down the machines, they have to make two different settings that are basically proportional to each other. There is formula that they can use, but it is really a proportion. So, if they understood how to use proportions, they didn't have to remember the formula. Participants got very good at direct proportions.

Comments:

- The instructor used an overhead projector to advantage for this class. She was able to face the class and work through problems like a chalkboard. It worked very well. Participants would talk about a theory or problem, do some examples and then solve a problem together. This seemed to help participants get the idea together. Because the instructor faced the participants, they could interact easily and get an understanding of what was clear, and what needed more explanation.
- For most participants, it takes quite a bit of practice to keep direct and inverse variation clear.
- The class spent about 4 or 5 sessions on proportions.

Activity: IV. Percents

1. Visualizing and understanding the concept of percent
2. Percent computation review
3. Relationships between fractions, decimals, and percent
4. Using a proportion to solve percent problems
5. Other ways of solving percent problems
6. Percent applications on the job and in everyday life

Resources and Materials Needed:

- Base ten blocks
- # # "*Focus Student Activity 39.1*" Visual Mathematics, Course 1, "Modeling Percent," The Math Learning Center, 1994. # # (Copy of this with Basic Math curriculum.)
- # # "*Focus Student Activity 39.2*" Visual Mathematics, Course 1, "Modeling Percent," # # (Copy of this with Basic Math curriculum.)
- # # "*Focus Master A (Situations)*" Visual Mathematics, Course 1, "Modeling Percent," # # (Copy of this with Basic Math curriculum.)
- # # "*Focus Master B*" (10 x 10 grid paper) Visual Mathematics, Course 1, "Modeling Percent," # # (Copy of this with Basic Math curriculum.)
- # # "*Activity Sheet VII-I-A*" Math and the Mind's Eye, The Math Learning Center, 1991. # # (Copy of this with Basic Math curriculum.)
- "*Activity Sheet VII-I-B*" Math and the Mind's Eye.
- # # "*Activity Sheet VII-I-C*" Math and the Mind's Eye. # # (Copy of this with Basic Math curriculum.)
- # # "*Activity Sheet VII-I-D*" Math and the Mind's Eye. # # (Copy of this with Basic Math curriculum.)
- # # Figures from Math and the Mind's Eye, Unit VII, Activity 1 ("Introduction to Percentages") and Activity 2 ("Fractions, Decimals & Percents") # # (Copy of this with Basic Math curriculum.)
- # # "*Percent Means per Hundred*," Mary Mayfield, Instructor, Lane Community College, 1996. # # (Copy of this with Basic Math curriculum.)
- # # "*Quick and Easy Everyday Percents*" Mary Mayfield 1996. # # (Copy of this with Basic Math curriculum.)

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- "Equal Fractions, Decimals, and Percents" Have Fun with Math, Louis Grant Brandes, J. Weston Walch, 1988.
- "Lineup with Fractions, Decimals, and Percents" Have Fun with Math.
- ## "Percent Problem Chart" Have Fun with Math. ## (Copy of this with Basic Math curriculum.)
- ## "Mystic Square with Percent" Have Fun with Math. ## (Copy of this with Basic Math curriculum.)
- ## "Solving Percent Problems" Mary Mayfield, 1978, revised 1997 ## (Copy of this with Basic Math curriculum.)
- "The Percent Equation" Decimals, Fractions, Ratios, and Percents, (part of the Smart Solutions series), p. 164-165, New Readers Press, Syracuse, NY, 1995.
- "Solving a Percent Equation" Decimals, Fractions, Ratios, and Percents, (Smart Solutions series), p. 166-167, New Readers Press, Syracuse, NY, 1995.
- ## "Word Problems with Percents and Proportions" Mary Mayfield, 1996. ## (Copy of this with Basic Math curriculum.)
- ## "Work-Related Math (A)" Mary Mayfield, 1996. ## (Copy of this with Basic Math curriculum.)
- "Work-Related Math (B)" Mary Mayfield, 1996.
- ## "Work-Related Math (C)" Mary Mayfield, 1996. ## (Copy of this with Basic Math curriculum.)
- ## "Work-Related Math (D)" Mary Mayfield, 1996. ## (Copy of this with Basic Math curriculum.)
- ## "Work-Related Math (E)" Mary Mayfield, 1996. ## (Copy of this with Basic Math curriculum.)
- ## "Percent Computation Review" Mary Mayfield, 1996 ## (Copy of this with Basic Math curriculum.)

Description of Instruction:

NOTE: As with the previous two activities, the materials and activities for this review of percents was drawn mostly from the **Basic Math curriculum (Activity VII: Working with Percents)**. Resources listed above are the ones used by this instructor and a brief summary of instruction is given below. For additional resources, activities, ideas or comments, please refer to the Basic Math curriculum.

1. Instructor began with: What does a percent mean? Percent means hundredths (how many out of a hundred). She also compared percents to fractions and decimals. Fractions are very

- flexible (any quantity can be compared to any other quantity). Decimals have to be so many 10ths, so 100ths, 1000ths, etc.. Percent is so many 100ths.
2. Instructor used overhead transparencies of "*Focus Student Activity 39.1*" and "*Focus Student Activity 39.2*." The participants had their own copy for reference or to make counting squares easier, etc. These worksheets were used as a quick group review. Participants called out answers; and were sometimes asked to state their reasoning. The instructor helped to clarify any questions. For "*39.1: i, j, k, and m,*" she gave them time to think about their approach and their answer; then continued with the group discussion.
 3. Instructor handed out "*Percent Means per Hundred*" as a reference.
 4. Instructor used overhead transparencies of "*Activity Sheet VII-1-A*" and "*Activity Sheet VII-1-B*" to continue group discussion of percents, especially focusing on approach to the problem. For example, 'How would you decide what percent this represents?' or 'How would you decide how to represent this percent?' "*Activity Sheet VII-1-C*" and "*Activity Sheet VII-1-D*" were given as homework. At the beginning of the following class, participants shared answers and instructor led a discussion about participants' thinking/problem-solving process on some of the more challenging problems.
 5. Instructor talked a bit about common percents (50%, 10%, 25%, etc.) and how to figure them easily. She also talked about how to use these percents in combination to easily figure other percents. (For example, 35% can be thought of as 25% plus 10% - or - as just over one-third, depending on the accuracy of the answer needed and how easy the numbers are to work with.) She used "*Quick and Easy Everyday Percents*" to give participants practice using common sense and their understanding of basic percent to solve problems, rather than having to use pencil and paper. The instructor's goal was for participants to understand how to approach and solve problems in the simplest way -- a way that made sense, rather than always relying on rules.
 6. The instructor briefly talked about doing conversions between fractions, decimals, and percents. She discussed with participants the steps shown for each process (converting percents to decimals, percents to fractions, etc.). As she talked, she worked examples on the overhead or the white board. The worksheets "*Equal Fractions, Decimals, and Percents*" and "*Lineup with Fractions, Decimals, and Percents*" were used for practice. Participants did a few problems on each with instructor monitoring and answering questions. For homework, the instructor asked them to do enough (choosing at least some that looked most challenging) problems that they felt comfortable with their understanding. At the beginning of the next class, the instructor led a discussion of questions and process, also providing the answers to help participants check for mistakes.
 7. Instructor taught participants how to use the skills learned with proportions as a way to solve percentage problems. She gave them "*Solving Percent Problems*" and worked through some sample problems. She then asked them to work problems from "*Percent Problem Chart*," using this method. Some participants remembered (more or less clearly) doing percent problems by multiplying or dividing. Some felt unsure about using proportions to solve percent problems. The instructor asked them to do a few problems using proportions just to

check it out; then she demonstrated with examples how she would work through a problem using the more traditional way.

She suggested that they first write out the problem in the form: ____% of ____ is _____. Next, if they knew the percent, they should convert it to a decimal and translate what they had written to a math statement: ____ $\times$ ____ = _____. Then, if they have the two numbers to multiply, they should multiply. If not, they should divide the number they have on the right of the equal sign by the number they have on the left of the equal sign. Participants who wanted to use this method of figuring percents used pages 164-167 from Decimals, Fractions, Ratios, and Proportions for instruction and practice.

Participants then worked problems in whatever way was comfortable and consistent for them.

8. The remainder of the time spent on this unit was review and word/work problem practice. "Percent Computation Review" helped anyone who needed to have further practice on all the aspects of percent the class had worked on. "Word Problems with Percents and Proportions" was an introductory practice in solving word problems with percents (and reviewing proportions as well). The "Work-Related Math" series (A-E) has a variety of customized math situations that use skills from basic computation through percents. Problems were assigned from these worksheets to
 - a) match participant skill level and need for practice, and
 - b) give participants a chance to work with many of the math-related situations that come up in this company.

Performance Assessment Methods and Results:

- Learning to use percentages with a common sense approach worked reasonably well, although participants needed much more practice than anticipated to really be comfortable with it. Working percents in their head was new to most of the participants. Participants found working with percents confusing, so the instructor tried to help them improve their "percent sense" as possible. This took more explanation and individual attention than anticipated. The concepts needed to be taught rather than just reinforced.

Comments:

- Throughout the class, the instructor emphasized process. She wanted participants to become aware of their own problem-solving processes. They needed practice at verbalizing their thinking, so that others could understand it. They also needed to experience other participant's approaches to problem-solving. Frequently, she asked: How did you do that? What was the first thing you did?
- The class spent 8 sessions on percentages. This was significantly more than the instructor anticipated. However, participants definitely wanted to improve their skills. And there were many applications of percent in their work situations: figuring percent of reject, percent of overweight, percent of pay for incentive bonus plan, etc.
- As noted in the Basic Math curriculum, this instructor's bias is toward teaching participants to solve percent problems using proportions. Some reasons for this bias:

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- a) When using the traditional approach to solving percent problems, participants often forget when to multiply/when to divide and which numbers to multiply or divide.
- b) Most participants seem to work well with proportions and use them successfully to solve percent problems.
- Not everybody did every worksheet. How many each person did depended on their skill level, on their speed, and so forth. The worksheets were available if participants needed them.

Activity V: Exponents, Roots, and Scientific Notation

1. Participants reviewed and computed exponents and roots, with and without a calculator.
2. Participants expressed standard decimal numbers in scientific notation and converted numbers in scientific notation to standard decimal numbers.
3. Participants computed with numbers in scientific notation.
4. Participants used a scientific calculator to do some work with numbers in floating decimal, fixed decimal, scientific, and engineering mode.

Resources and Materials Needed:

- # "Notation Worksheet 3a (Using Exponents)" GED Priority Math, Mary Ann Schmidt, Massachusetts Department of Education, 1989.
- # "Notation Worksheet 4a (Square Roots and Squares)" GED Priority Math.
- "I Have, Who Has?" Teacher's Resource Guide: Algebra and Geometry, (Smart Solutions series), p. 45, Jan Phillips and Kathy Osmus, New Readers Press, Syracuse, New York, 1996.
- Scientific calculators
- # "Scientific Notation" Mary Mayfield, Instructor, Lane Community College, 1997.
- # "More Practice with Scientific Notation" Mary Mayfield, 1997.
- # "Time to Explain Yourself" Mary Mayfield, 1996.

Description of Instruction:

1. The instructor first reviewed exponents and square root in a group discussion based on some examples she wrote on the white board. Then, for individual practice and to check understanding, participants did the "Using Exponents" and "Square Roots and Squares." They also did "I Have, Who Has?." For this activity, participants use cards reading "I have 30, who has 3^2 ?" or "I have 9, who has $\sqrt{144}$?" etc.
2. The instructor led a discussion about how to do powers and roots on the calculator. At this point, participants discussed roots other than square root. However, the instructor told participants that in this class, they'd have little reason to use those. They did a brief practice with the instructor just calling out problems (95 squared, the third root of 125, etc.) and participants calling out answers when they got them--and also sometimes having to explain their process.
3. The instructor introduced scientific notation by reviewing the decimal (base 10) system and the powers of 10 on which it is based. She gave participants "Scientific Notation" and talked with them about the concepts as they worked through it. She reviewed and demonstrated expanded notation and computation with exponents first using only positive exponents. Then

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when they had established some patterns or rules, she applied them to numbers with negative exponents as well, trying to give participants some sense of the reasonableness and/or consistency of the system. For instance, she would show that (using the rules that participants already understood) $100 \times .001$ is the same as $100 \times 1/1000$ is the same as $10^2 \times 10^{-3}$.

Participants worked through "*Scientific Notation*" and "*More Practice with Scientific Notation*" mostly in class with instructor assistance as needed.

4. Once participants had completed "*Scientific Notation*" and had some experience and some understanding working with scientific notation, the instructor guided them in discussion of and experimentation with the floating decimal, fixed decimal, scientific, and engineering modes of the scientific calculator. Then participants used the calculator to check their answers on the last page of "*Scientific Notation*" and as instructed on "*More Practice with Scientific Notation*."
5. "*Time to Explain Yourself*" was an opportunity for participants to review the skills and concepts in this unit and to practice explaining their knowledge and their thinking. Rather than have participants work through this as a worksheet, the instructor had them work in pairs. She assigned each pair 2 or 3 different types of problems (for example, one pair might work on #1, #4, and #7). The pairs were to do what was asked (for example, draw a representation of $3x^2$ or calculate 7^5 on their calculator). Then the pairs were to discuss how best to explain the process, possibly practicing the explanation on each other. The instructor usually didn't assign question #3 (Explain the difference between $3x^2$ and $(3x)^2$). Instead, she asked this question of the whole group after both $3x^2$ and $(3x)^2$ had been explained and demonstrated.

Performance Assessment Methods and Results:

- This group was reasonably familiar with a lot of math concepts such as squares, square roots, etc. While each person didn't have all the information at his fingertips, everyone had enough background so that an instructor-led discussion would actually gather from the participants most of the information necessary to introduce a topic. Participants were comfortable with each other and so would volunteer information, ideas, or questions. This was a good way for the instructor to judge the skill or understanding level of the group and made for an interactive, participatory beginning that usually went fairly quickly. The instructor, of course, supplied missing information or emphasized points she thought were particularly important.

Comments:

- The instructor worried a bit that "*I Have, Who Has?*" would seem too simplistic or juvenile for this mostly over-40 group. In fact, they seemed to enjoy it.
- Scientific (and engineering) notation were introduced into this class because a maintenance worker showed the instructor some instructions and information materials for some of their computerized equipment in which some of the numbers were written in one of these notations. The purpose of this unit of instruction was primarily to give

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participants some information and experience that they could draw from if they needed to understand materials using such notations.

- Participants were quite comfortable with positive exponents, but most had no familiarity with negative exponents. They, in fact, found it to be a rather incomprehensible concept. They also were mostly unfamiliar with expanded notation, although they found that easy to understand (less easy, however, when it expanded into decimal fractions and negative exponents).

Activity VI: Angles, Lines and Triangles including the Pythagorean Theorem

1. Participants worked with the properties of angles, lines and triangles.
2. Participants used hands-on activities to clarify concepts related to the properties of angles, lines and triangles.
3. Participants learned and worked with the Pythagorean theorem.
4. Participants worked with practical applications of the Pythagorean theorem.

Resources and Materials Needed:

- "Angles" pp. 6-29, Number Power 4: Geometry, Robert Mitchell and Donald Prickel, Contemporary Books, Chicago, Illinois, Copyright 1989.
- "Triangles" pp. 34-53, Number Power 4: Geometry.
- Pages 56-61 and pp. 148-149, Number Power 4: Geometry.
- # "How Many" Math Intersections: A Look at Key Mathematical Concepts, Blackline Masters, p.45, David J. Glatzer and Joyce Glatzer, Dale Seymour Publications, Palo Alto, CA, 1990.
- # "Review of Geometrical Terms and Figures," Mary Mayfield, Instructor, Lane Community College, 1997.
- Scissors and plenty of paper
- # "Plane Geometric Figures" Mary Mayfield, 1994.
- # "Practice with Pythagoras" Mary Mayfield, 1996.

Description of Instruction:

1. "How Many?" made a good warm-up, introductory activity. The instructor gave the participants a few minutes to individually decide on answers to as many of the questions as they could. The instructor emphasized that it didn't matter how much or how little of the material they knew--that this was supposed to be fun, just a way to get the terms out for discussion. (In one class where this was used, participants were unfamiliar with many of the terms and needed a little extra reassurance before they could treat it lightly enough to have fun.) After a few minutes, the instructor asked participants to pair up, compare answers, and to "put their heads together" to see if they could determine any more answers. Then the whole class discussed each question, with the instructor giving definitions as needed and extending some of the concepts when she thought it would be helpful.
2. Each participant had a Geometry book and the class followed it fairly closely for the work with angles and triangles. The instructor tried to minimize vocabulary intimidation brought on by sudden exposure to lists of unfamiliar, multi-syllable words. She did this by concentrating on concepts and emphasizing that the vocabulary was just a way to talk about

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the concepts. (For instance, angles can be described as acute, obtuse, right, complementary, supplementary, vertical, adjacent, interior, exterior, alternating, etc. The instructor tried to focus on the relationships that inspired this naming and on how understanding the relationships could be useful.)

3. This unit was done mostly as a group discussion activity. In one course, the instructor followed through the book, talking about the concepts in each part and discussing with the group approaches and answers to selected exercises. In another course, the instructor gave participants a list of terms they would be discussing. Then, she drew on the white board a variety of figures (lines, angles, triangles, other plane figures) that would serve as "memory-joggers" or "discussion-focusers." (She purposely did not make a one-to-one correspondence between the list of terms and the drawings. She wanted the drawings to serve as visual reminders, but she also wanted participants to have to think about how things were related.) After the class had an opportunity to think briefly about these concepts, she went through the list of terms with them, discussing related terms as a group, adding in additional concepts, answering questions, expanding ideas, etc.
4. The following activity was used to show the relationship between the angles in a triangle and to illustrate supplementary angles. This activity could also be used for quadrilaterals. Each participant was given scissors, some sheets of paper, and the following instructions:
 - Cut out a fairly large, random triangle from one of your sheets of paper. Do not use more than one of the edges of the paper for a side of the triangle. (*This discourages an abundance of right triangles.*)
 - Next cut the triangle in 3 parts so that each part contains one complete angle of the triangle.
 - Arrange the angles with vertices together and forming one solid figure. What do you notice?
 - Try again with a differently shaped triangle. Do you get the same result?
 - What do you learn from this?
5. The instructor reviewed definitions of triangle, right angle, right triangle, and hypotenuse. Then she used small blocks to lay out a 3-4-5 triangle and built squares on each side with more blocks (so there was a 3×3 square, a 4×4 square, and a 5×5 square). She gave participants the opportunity to point out anything they noticed about the situation. She noted, if someone else didn't, that the total blocks on the 2 sides equaled the block on the hypotenuse. She represented that as $a^2 + b^2 = c^2$ and said that it was true of all right triangles. She drew some triangles (noting measurements of 3 sides) on the overhead and asked if they could use this principle (the Pythagorean Theorem) to tell if these were right triangles. Then she drew right triangles (giving measurements for 2 sides, but not hypotenuse) and asked how they might find the length of the hypotenuse. They worked through a couple of examples. Then she drew another right triangle (giving measurements for the hypotenuse and one side) and asked how they would find the length of the other side. The group worked through some other examples together.
6. The instructor explained that pages 56 and 58 in Geometry were written explanation and examples of what participants had just gone through. She had them practice with the

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problems on pages 57, 59, 60, 61 while she monitored and answered questions. Then she gave them "*Practice with Pythagoras*" for homework.

7. At the beginning of the next class session, participants discussed process, answers, and questions from that homework.

Comments:

- Math Intersections activities make participants think about how things fit together and why. They are good for review and for introducing new concepts. Also, once participants get over some possible initial panic because of unfamiliar terms, they enjoy this activity. There are several other Math Intersections activities similar to this one ("*Adjectives*," "*Analogies*," "*Opposites*," and more) that the instructor has used for review or warm-up activities quite successfully.
- This class did not do any work with a protractor. Participants seemed to understand the concept of measuring angles and didn't think they needed to practice it.
- The participants liked working with angles and triangles because it seemed so practical and concrete. This was a welcome contrast with percentages and proportions. Participants needed to understand and be able to work with percentages and proportions, but they seemed abstract and hard to visualize.
- The class learned how to use similar triangles and proportions to determine the height of objects (when that was difficult to measure directly). Participants also learned how to use proportions to determine the distance across canyons or bodies of water that are also difficult to measure directly. This was a new concept to the participants--one they thought might be useful in making estimates in many contexts, even when fishing or similar leisure activities.

In future classes, the instructor would like to set this up as a hands-on activity. She would create a (probably imaginary) unmeasurable object in the classroom or find an appropriate situation outdoors. Then participants would determine how they could set up pairs of similar triangles to measure a particular point-to-point distance across that barrier.

- Participants felt they learned concepts and skills that would be useful, not only on the job, but also in their personal life.
- Most of two sessions were spent on the Pythagorean Theorem.
- The participants enjoyed working with this because it seemed so practical and concrete. They could see where they would apply this concept and how to put it into practice immediately. Because of the practical nature of the subject, working with it went more smoothly than some of the earlier subjects.

Activity VII: Computing with Measurements

1. Practiced efficiently and accurately computing with measurements
2. Learned to use unit conversion (also called dimensional analysis) to convert values from one unit to another

Resources and Materials Needed:

- Number Power 4: Geometry, pp. 74-77, Robert Mitchell and Donald Prickel, Contemporary Books, Chicago, Illinois, Copyright 1989.
- Pre-GED Mathematics Exercise Book B: Fractions to Geometry, p. 63, Glencoe/McGraw-Hill, Westerville, OH, 1994.
- # "Unit Conversion or Dimensional Analysis" from: ONE to Our Rescue, Rose Steiner, (a packet of materials created for a workshop in 1996) Billings Adult Education Center, Billings, MT.
- # "Practice with Unit Conversion/Dimensional Analysis" series of worksheets, Mary Mayfield, 1996.

Description of Instruction:

1. The instructor assigned pages 74-77 of the text as homework, asking the participants to read the examples and instructions and then work at least all the English measurement problems. Given the skills and understanding of this group, she thought this would be just a quick review and she gave no lecture or preview information. Reviewing their efforts at the beginning of the next class, she discovered that they had either not read or not really understood the explanations given in the text. Participants were able to do the problems, but they converted each measurement to a one-unit measurement (as converting 7 ft. 3 in. to 87 in.), did the computation, and then converted back to a two-unit measurement (as converting 11 ft. to 3 yd. 2 ft.)

The instructor then showed them the process for doing the computations without doing those conversions. She emphasized that both methods work just fine, but that computing without doing the conversions is quicker and, because it involves fewer steps, has fewer opportunities to make mistakes.

Participants then used problems on p. 63 of Pre-GED Mathematics Exercise Book B to practice using this method of computation.

2. The instructor introduced unit conversion/dimensional analysis as a method for changing values from one unit to another. She gave participants "*Unit Conversion or Dimensional Analysis*" as a reference. She talked a little about the technique, then used the technique to do a simple conversion and a more complicated one. She then went through several examples with the group. (One way to use the examples in "*Unit Conversion or Dimensional Analysis*" is to have participants cover the written-out example and then work them through the steps of setting up each example, comparing the final result with what was given.) Initially, the instructor did examples herself, explaining each step and asking for class input

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such as "How many feet in a mile?" Then, she would begin asking the class how to set up the problem: "What should I write first?" or "Where should I write this unit?" Finally, she had them try a few on their own. Once they seemed to be "getting it," participants worked on "*Practice with Unit Conversion/Dimensional Analysis(1)*" with instructor assistance as needed. They finished this worksheet as homework, discussed it in the following class, and worked a little on "*Practice with Unit Conversion/Dimensional Analysis(2)*" (which they completed as homework with discussion the following class period). After that, the instructor would sometimes present at the beginning of class, one or two problems from their work environment for them to continue to practice this technique.

Performance Assessment Methods and Results:

- Computing with measurements was expected to be a review activity, not an assessment activity, but surprises happen. The instructor gained important information about participant abilities and approaches that she used to fine-tune instruction. She also gained information helpful for future planning.

Comments:

- The instructor had not intended to spend much time computing with measurement. Some additional time was spent instructing in a second way of doing these computations. Still, participants already knew a way of doing the computations that worked and they caught on quickly to the new method, so they didn't spend a lot of class time on it.

In the future, the instructor would definitely introduce this more carefully, explaining the computation method she would suggest before assigning practice problems. She also might spend more time on it because it has the potential for eliminating errors on the job. Better yet, she would include it in the Basic Math curriculum.

- Unit conversion (dimensional analysis) can be used for any unit conversion problem, but is most useful in complex conversions (those that would require 3 or more steps in the usual conversion process). It is also helpful for participants who have trouble remembering when to multiply or when to divide when doing conversions. It is a great technique (and really quite simple), but participants need lots of time to think, process, and practice before they really "get it."
- Essentially every participant in this instructor's classes initially resisted learning unit conversion. They either thought it too trivial because in applying it to simple conversions their old methods worked just as well -- or -- they thought it too complicated or without significant application. However, one of the participants most resistant to learning it--who had strongly indicated that it wasn't worth his time--later picked it up, was very happy with it, and found numerous application for it in his maintenance work.
- Note that set-up for problems using unit conversion may vary depending on what participants remember about unit equivalence's. (For instance, to convert from inches to yards, one person might use 12 inches per foot and 3 feet per yard while another person would use 36 inches per yard.) Also note that it is important to provide both plenty of demonstrated examples and plenty of practice. This may be especially true when working

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with square or cubic units. An additional advantage when using the technique with square or cubic units is that it will work quite well if you only remember basic equivalence's. You don't have to remember there are 144 inches in a square foot, only that there are 12 inches in a foot.

- This instructor did not use the practice problems on the last page of "*Unit Conversion or Dimensional Analysis*," although she did incorporate some of them into her worksheets. She felt that participants in her classes needed more practice paced differently than this practice page allowed.

Activity VIII. : Perimeter, Area and Volume

1. Taught perimeter as distance around straight-sided figures
2. Worked with the area of triangles, rectangles, squares, trapezoids, parallelograms
3. Worked with the circumference and area of circles
4. Worked with the surface area
5. Worked with the volume of cubes, rectangular solids, cones, cylinders, spheres, pyramids
6. Worked with the application of area and volume formulas

Resources and Materials Needed:

- Number Power 4: Geometry, Robert Mitchell and Donald Prickel, Contemporary Books, Chicago, Illinois, 1989. ***or***
Algebra and Geometry, (Smart Solutions series), Jerry Howett, New Readers Press, Syracuse, New York, 1996.
Teacher's Resource Guide: Algebra and Geometry, Jan Phillips and Kathy Osmus (Smart Solutions series), New Readers Press, Syracuse, New York, 1996 . ***or***
"Unit 9: Geometry" Number Power Review: Whole Numbers to Algebra, Robert Mitchell, Contemporary Books, Inc., 1989.
- Get It Together: Math Problems for Groups, Tim Erickson, Equals, Lawrence Hall of Science, Berkeley, CA, 1989.
- "Finding Perimeters and Areas from Grid Measurements" Have Fun with Math, J. Weston Walch, Publisher, 1988.
- # "Finding the Area of Old Shadowface" Have Fun with Math.
- "Fold and Tape Geometry" Trivia Math: Pre-Algebra, p. 26, Catherine Pedersen, Creative Publications, Mountain View, CA, 1988.
- "Perimeter, Area, and Volume," GED Priority Math, p. 16, Mary Ann Schmidt, Massachusetts Department of Education, 1989.
- # "Review" Mary Mayfield, Instructor, Lane Community College, 1996.
Questions #20-23 were taken from Trivia Math: Pre-Algebra, to use for review here.
- # "Formulas and Definitions" Mary Mayfield, 1996.
- # "Volume," Worksheet 7, GED Priority Math.
- Trivia Math: Pre-algebra, Katherine Pedersen, Creative Publications, Mountain View, CA, 1988.

Description of Instruction:

1. During the course of the class, the instructor used a number of the "cooperative logic" problems from *Get It Together*. These problems provide an opportunity for participants to work together, to problem-solve, to pay attention to detail, to use mathematical thinking and reasoning, to work with mathematical concepts (though not all the problems involve mathematics). For this unit, problems from the "*Stick Figures*," "*Pattern Blocks*," or "*Polygons*" sections seemed most appropriate, but there are many other choices depending on the skill of the group and the concepts to be explored.

For these activities, each person gets a clue (or sometimes 2 if the group is small). Then:

- "When you get your clue, you may look only at your own clue. You may not look at anyone else's."
- "You may share your clue by telling others what's on it, *but you may not show it to anyone else.*"
- "If you have a question you must check with your group first. If your group agrees that everyone has the same question, you may all raise your hands and the teacher will come."

Despite the simplicity of the rules, participants often have trouble following them.

2. The class had already discussed geometric figures and definitions. Now the instructor gave participants "*Plane Geometric Figures*" as a review or reference. It also organizes the quadrilaterals from most general to most specific--which gave an opportunity to talk about a square as a special case of rectangle, a rectangle as a special case of parallelogram, etc.
3. The class did most of the instructional work on geometrical formulas using one of the texts listed above. The instructor chose the text that she felt best fit the skills and needs of the participants in that particular class. She introduced each new concept or formula with drawings, explanations, hands-on models, and/or examples, and she supplemented the text activities with additional work to consolidate and/or extend skills.
4. "*Finding Perimeters and Areas from Grid Measurements*" and "*Find the Area of 'Old Shadowface'*" gave participants an opportunity to practice their use of perimeter and area formulas either in class or as homework. The instructor liked these worksheets because they are clear and easy to understand, yet they require attention to detail, understanding how to work with a grid, and some thought about what measurements are needed.
5. To begin looking at volumes of solid figures, the class did a warm-up activity called "*Fold and Tape Geometry*." The instructor asked pairs of participants to work with pieces of paper, scissors, and tape. They are to create some of the shapes shown on the activity page. She assigned (by lottery) 2 shapes to each pair, but they could try others if they finished and wanted to. Some shapes were very easy, but some were quite difficult to figure out. This was an enjoyable and interesting activity that led to discussion of characteristics of the figures as well as how participants approached the problem.
6. The instructor preferred to keep instruction (and rules, formulas, etc.) as simple as possible, and wanted participants to understand relationships between various concepts. For those reasons, she taught formulas for volume of cylinders and prisms (solids with 2 parallel

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congruent bases and rectangles or parallelograms for faces) as "the area of the base times the height." For the same reasons, she taught volume of cones and pyramids as " $\frac{1}{3}$ times the area of the base times the height." This worked quite well.

7. The instructor usually gave participants "*Formulas and Definitions*" when they had finished the work on individual formulas and were beginning to review. The handout offered a review of formulas and definitions as well as a reference to use as needed.
8. Sometimes the instructor used the worksheet "*Volume*" as it stands, but often she waited to use it until participants had studied both volume and surface area. Then she had them figure both volume and surface area of the figures on the worksheet.
9. Often the instructor asked participants to estimate answers to certain problems before actually computing the answers. She then discussed with the group what the approach (or approaches) to the problem was, what patterns they looked for to make estimation easier, when they might use estimation instead of computation, etc. She also reminded them that estimation can also be a quick double-check on whether a computed answer is reasonable.
10. "*Review*" is a compilation of problems from a variety of sources that gives participants an opportunity to review most of the skills learned so far. "*Review*" was given as homework; in the next class, the class discussed the processes and thinking that participants used and any questions they had. Questions 20-23 were taken from Trivia Math: Pre-Algebra. They were useful questions because most of them require more than one step to solve and require applying more than one formula or concept.

Performance Assessment Methods and Results:

- The instructor looked at the homework brought to class.
- The instructor interacted with participants as they brought questions and problems with the worksheets to class.
- Generally participants did well when working with areas, perimeters and volumes, although some had some difficulty remembering the formulas. Many of the participants hadn't used these formulas in a long time. Still, they understood the processes and saw the usefulness of the information.

Comments:

- The three books listed first in the resource section above all have good, clear, informative sections on perimeter, area, and volume. The instructor successfully used each in different situations. The section in Number Power Review (pp. 218-225) is a short, clearly written section that works well for a quick review for those who just need a bit of a brush-up.
- A lot of the class was pencil and paper, and the work the instructor did was customizing the materials to make them more appropriate for the skills and needs of the participants. She also added workplace examples whenever she could. However, there are fewer workplace examples included here than in some other sections. This is partly because

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participants felt that they could see the applications for these skills and wanted to move on to learning other skills. Also, the instructor did not have as many opportunities to collect workplace examples for this material. The participants did bring in problems occasionally that the class discussed and worked. For instance, several participants wanted to find the volume of the silos that hold their raw material. The silos are cylinders above an inverted cone. The instructor asked them for possible dimensions, drew the silo on the white board, and participants worked through the steps to determine volume of material at different levels in the silo.

- Participants never really got comfortable with the step by step process for estimation, but the instructor still thought it was a valuable skill for them to know about. This is an area that probably needs more attention in future classes.
- Throughout the class sessions, the participants were encouraged to ask questions and to make sure the concepts were clear as the problems were discussed. A lot of conversation created a relaxed atmosphere, and participants were less and less afraid to ask questions or reveal that they didn't understand something.
- This class did quite a bit of homework. Often the instructor would suggest that participants finish at home some work they had begun in class. Then, they would review that homework at the beginning of the next class. Participants did whatever homework their time, energy, and skills allowed. There was never any pressure about whether homework was done. The instructor adjusted the day's agenda as needed to accommodate what participants had done, what they had understood, etc.

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Activity IX: Basic Algebra Concepts

1. Illustrated signed numbers with number line and real life examples
2. Computed with signed numbers
3. Used the order of operations to do sequences of calculations
4. Compared how a 4-function calculator and a scientific calculator handle such sequences
5. Wrote and interpreted algebraic expressions using variables
6. Used substitution to evaluate algebraic expressions and formulas
7. Simplified algebraic expressions

Resources and Materials Needed:

- Algebra and Geometry, (Smart Solutions series), Jerry Howett, New Readers Press, Syracuse, New York, 1996.
- Teacher's Resource Guide: Algebra and Geometry, Jan Phillips and Kathy Osmus (Smart Solutions series), New Readers Press, Syracuse, New York, 1996. (This text/workbook is a clearly written presentation of pre-algebra and basic geometry skills and the teacher's guide has many good supplemental activities. One useful activity, # "Using Algebra to Describe Patterns" from the teacher's guide is included with activity)
- *or*
- Algebra, James T. Shea (Working with Numbers series), Steck-Vaughn Company, Austin, TX, 1995. (This is a very thorough, well-organized text/workbook with lots of practice problems, including topics from writing algebraic expressions through working with quadratic equations.)
- *or*
- Number Power 3: Algebra, Robert Mitchell, Contemporary Books, Inc., Chicago, IL, 1988.
- Trivia Math: Pre-Algebra, Katherine Pedersen, Creative Publications, Inc., Mountain View, CA, 1988.
- "Set-up Problems" Worksheet 1, from the Algebra section of GED Priority Math, , Mary Ann Schmidt, Massachusetts Department of Education, 1989.
- # "Order of Operations" Mary Mayfield, Instructor, Lane Community College, 1996.
- # "Number Tricks" Adapted from "Tricks and Why They Work," Enrichment Mathematics for the Grades, National Council of Teachers of Mathematics, Washington, 1963. Mary Mayfield, (Adaptation, 1997.)
- Manipulatives: blocks or other objects or 2 different colors or 2 different sizes, shapes, etc.

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- "Activity 1: The Handshake Problem," Math and the Mind's Eye, Unit 1: Seeing Mathematical Relationships, The Math Learning Center, Salem, OR, 1988.
- "Activity 1: Toothpick Squares: An Introduction to Formulas," Math and the Mind's Eye, Unit IX: Picturing Algebra, The Math Learning Center, Salem, OR, 1993.

Description of Instruction:

NOTE: This instructor taught various combinations of algebra and geometry skills to three different groups of participants from the same company. Each of those classes used one of the above texts and the course of instruction followed the text pretty closely. Therefore, what is included here will primarily be supplemental materials or activities.

1. Most class sessions opened with a warm-up problem or two. Usually these were questions involving skills participants already had but presented in a rather unique problem-solving context. Most of these problems came from Trivia Math: Pre-Algebra.
2. At the first or second class meeting, participants worked on "Set-up Problems." The instructor first had them scan through the worksheet (particularly the multiple choice answers) to see if there were symbols or terms they didn't understand. Some possibly unfamiliar symbols are parentheses for grouping and multiplication, slant line or fraction line for division, brackets, etc. The instructor explained these symbols, then had participants work some of the problems in class. When participants seemed to have a sense of what they were doing, she suggested that they do the rest as homework.
3. "Using Algebra to Describe Patterns" was a great activity. The instructor had participants work in pairs or threes. There was quite a bit of discussion between partners and a lot of "mulling over" of what the patterns might be and how the equations should be grouped. Once they had grouped, participants checked their groupings with those on the activity sheet before going on to write the algebraic equation. Writing the equation was done partly as a whole group activity because some participants weren't quite sure what to do. Once they got started, though, the algebra started to "flow," and to make sense.
4. The instructor used the acronym PEMDAS for the rules of order of operations.
 - P = Parentheses
 - E = Exponents
 - MD = Multiplication and Division
 - AS = Addition and Subtraction

She had a hard time finding enough practice activities for learning order of operations, so she created several worksheets for that practice. "Order of Operations" is one of those.
5. "Number Tricks" was a way to build some understanding of algebra processes on something familiar. Before giving participants the worksheet, the instructor did a number trick with them (either the first one on the worksheet or another from her bag of tricks). If everyone came up with the same answer, then the discussion focuses on "why." If answers are different, participants double-check their work until they all, hopefully, have the same

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answer. (The answer should always be 1.) Participants may do the first number trick on the page at this point, if the instructor opened with a different one.

The instructor then modeled one or both of these first tricks with blocks. For instance, she might use a larger block to represent "the number" and a smaller block to represent each unit. Then "the number" is one larger block; adding 3 to it gives one larger block and 3 smaller blocks; doubling gives 2 larger blocks and 6 smaller blocks, etc. The instructor pointed out (if a participant doesn't point it out first) that the modeling ends up with as many units (smaller blocks) as the answer participants had determined.

Participants, in pairs usually, then had an opportunity to model the four tricks on the first page. The instructor monitored the process, offered suggestions, and answered questions. When everyone had modeled most or all of those tricks, she suggested using "x" to stand for "the number" and go through the process again. (This time "the number is "x"; adding 3 gives " $x + 3$ "; doubling gives " $2x + 6$," etc.)

Participants finish the work on the first page. On the next page, the instructor has them model what is given of the first trick, using both blocks and algebraic notation. She works with them to come up with a possible ending for that trick. Then she asks each of them to try to come up with a different ending for the same trick. When they have done that they compare notes and discuss what has to happen in order to end a trick. Participants then try to find endings for the rest of the tricks.

6. The instructor used either "*The Handshake Problem*" or "*Toothpick Squares*" or both as another visual and interactive way to present the use of algebraic expressions and formulas. "The handshake problem is used to illustrate the use of visual thinking in mathematical problem solving. In Part I, an expression is obtained for the number of handshakes if everyone in the classroom shakes hands with one another. This expression is evaluated in Part II." In "*Toothpick Squares*" participants form rows of squares with toothpicks. Participants investigate the relationship between the number of squares in a row and the number of toothpicks needed to form them. This leads to the introduction of algebraic notation and the use of formulas.

Performance Assessment:

- The instructor assessed performance by using review activities in the text(s).
- She also monitored the work participants did in class, paying attention not only to what they were able to do, but to the questions they asked, the comments they voiced, and mistakes they made.

Comments:

- "*Set-up Problems*" is a way to have participants work with mathematical expressions and focus on how problems are solved--in the context of fairly straightforward word problems and without the additional complications of variables. The following is a sample problem from that worksheet.

Course Title: Algebra / Geometry

"The average employee income at the Light Electric Company is \$320 weekly. There are 70 employees in the company. Which expression represents the company's yearly payroll?"

- (1) $70(\$320)(12)$
- (2) $70(\$320)$
- (3) $52(\$320)$
- (4) $52(\$70 + \$320)$
- (5) $70(\$320)(52)$

- The "*Number Tricks*" activity required quite a bit of instructor support. Participants weren't really comfortable with the modeling and didn't always recognize what was happening (what the pattern was, etc.). This is another activity that it is important for the instructor to be really comfortable with before presenting it, so as not to add further confusion for participants.
- Math and the Mind's Eye materials are wonderful. The two units mentioned here have a number of other interesting activities. There are many other helpful units as well.

Activity X: Working with Equations

1. Practiced and developed recognition of equivalent equations
2. Used a systematic, step-by-step approach to solve linear equations in one variable
3. Translated word problems into linear equations in one variable
4. Used linear equations to solve word/work problems

Resources and Materials Needed:

- Algebra and Geometry, (Smart Solutions series), Jerry Howett, New Readers Press, Syracuse, New York, 1996.
- Teacher's Resource Guide: Algebra and Geometry, Jan Phillips and Kathy Osmus (Smart Solutions series), New Readers Press, Syracuse, New York, 1996. (This text/workbook is a clearly written presentation of pre-algebra and basic geometry skills and the teacher's guide has many good supplemental activities)
- *or*
- Algebra, James T. Shea (Working with Numbers series), Steck-Vaughn Company, Austin, TX, 1995. (This is a very thorough, well-organized text/workbook with lots of practice problems, including topics from writing algebraic expressions through working with quadratic equations.)
- *or*
- Number Power 3: Algebra, Robert Mitchell, Contemporary Books, Inc., Chicago, IL, 1988.
- Activities for matching equivalent equations
- # "Words into Symbols" Mary Mayfield, 1996.
- # "What's That Number?" Have Fun with Math, p.168, Louis Brandes, J. Weston Walch, Publisher, Portland, ME, 1988.
- Resource Book for Algebra: Structure and Method, Book 1, Dolciani, Brown, and Cole, Houghton Mifflin Company, Boston, MA, 1986.
- How to Solve Word Problems in Algebra: A Solved Problem Approach, Mildred Johnson, McGraw-Hill, Inc., New York, NY, 1992. (# "Using Charts in Solving Problems" is included with this curriculum as an example of the materials available in this book.)

Description of Instruction:

NOTE: Each of the algebra classes used one or more of the above texts, and the course of instruction followed the text/s pretty closely. Therefore, included here are primarily supplemental materials or activities.

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1. The instructor used at least two different activities to help participants recognize that equations which may look very different may actually be the same equation in a different form. One of these activities was "*A Matching Game*" from *Trivia Math: Pre-Algebra*, p. 72. For another activity, she created a list of about 15 equations (most of which were equivalent to one other equation in the list; some to more than one). The instructions were: "Use your skills for operating on equations to decide which of these equation are equivalent to each other."
2. "*Words into Symbols*" gave participants more practice setting up equations. It is paced practice in that each of the three sections moves the participant a bit closer to being able to set up the equations on his/her own.
3. The instructor used "*What's That Number*" somewhat differently than it was designed to be used. She had participants write each statement as an equation. (For instance, "Twice the product of 6 and 5 is three times as great as this number" would become $2(6)(5) = 3x$).
4. The instructor taught the following plan for solving a word problem.

Step 1: Read the problem carefully a few times to gather information. Decide what numbers are asked for and what information is given. Making a sketch may be helpful.

Step 2: Choose a variable and use it with the given facts to represent the number(s) described in the problem. (Make note of what this variable represents.)

Step 3: Reread the problem. Then write an equation that represents the relationships among the numbers in the problem.

Step 4: Solve the equation and find the required numbers.

Step 5: Check your results with the words of the problem to be sure they make sense.
5. "*Using Charts in Solving Problems*" gives a systematic way to organize your thinking when setting up algebraic equations to solve word problems. This was a useful system for many participants.
6. The instructor had a hard time finding enough word problems with at least some connection to reality to give participants the practice they needed. To meet this need, she compiled several sets of problems.

Performance Assessment:

- Review activities in the text(s) were one way the instructor assessed performance.
- She also monitored the work participants did in class, paying attention not only to what they were able to do, but to the questions they asked, the comments they voiced, and mistakes they made.

Comments:

- The original plan for at least one of the algebra classes was that it would include work with solving systems of linear equations with two variables. For a number of reasons,

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including increased work load for some participants, the class disbanded without covering this material.

- Note that Resource Book for Algebra: Structure and Method, Book 1 has activities similar to "*Using Charts in Solving Problems*" for Cost-Value problems, Distance-Rate-Time problems, etc. These activities were all quite helpful and easy for participants to follow. This resource book has other useful materials as well.

Activity: XI. Working with Graphs

1. Interpreted charts and graphs accurately
2. Graphed linear equations in two variables, using a table of solutions
3. Graphed linear equations in two variables, using the slope and y-intercept
4. Collected data and graph data, found a line of best fit, created equation, used this information to interpret or predict

Resources and Materials Needed:

- Algebra and Geometry, (Smart Solutions series), Jerry Howett, New Readers Press, Syracuse, New York, 1996.
- Teacher's Resource Guide: Algebra and Geometry, Jan Phillips and Kathy Osmus (Smart Solutions series), New Readers Press, Syracuse, New York, 1996. (This text/workbook is a clearly written presentation of pre-algebra and basic geometry skills and the teacher's guide has many good supplemental activities. One useful activity, # "*Is a Picture Worth a Thousand Words?*" from the teacher's guide is included with this curriculum.)
or
- Algebra, James T. Shea (Working with Numbers series), Steck-Vaughn Company, Austin, TX, 1995. (This is a very thorough, well-organized text/workbook with lots of practice problems, including topics from writing algebraic expressions through working with quadratic equations.)
or
- Number Power 3: Algebra, Robert Mitchell, Contemporary Books, Inc., Chicago, IL, 1988.
- # "*Identifying Qualitative Graphs*" "Mathematics Teacher," September 1995
- Problem-Solving Experiences in Pre-algebra, Randall I. Charles and Carne S. Barnett, Addison-Wesley Publishing Company, Menlo Park, CA, 1992.
- # "*Notes About Graphs of Equations in 2 Unknowns*" Mary Mayfield, Instructor, Lane Community College, 1997.
- Algebra Experiments I: Exploring Linear Functions, Mary Jean Winter, Ronald J. Carlson, Addison-Wesley Publishing Company, Menlo Park, CA, 1993. (This book has 17 hands-on, data collecting experiments involving linear function. Materials for the experiments are easy to come by. The "Teaching Notes" are complete and easy to follow. The worksheets are well-written, using a clear step-by-step process to help participants through collecting data, finding an equation, and interpreting the data. # Part of one of these worksheets is included with this curriculum as a sample.)

Description of Instruction:

NOTE: Each of the algebra classes used one or more of the above texts and the course of instruction followed the text/s pretty closely. Therefore, what is included here will primarily be supplemental materials or activities.

1. "*Identifying Qualitative Graphs*" was a good warm-up activity to let participants have a little fun and give them some practice interpreting relationships expressed in line graphs. The instructor also used some similar activities from Problem-Solving Experiences in Pre-algebra.
2. The instructor had participants work on "*Is a Picture Worth a Thousand Words?*" after they had learned both graphing linear equations from solution tables and how to recognize the slope and y-intercept of the line from the equation. She gave them the worksheet asking them to match each graph with the correct equation. Participants were to try to find more than one way to do this, and to think about what would be the most efficient way to do this.

Participants usually approached it by trying out the values in the equations, usually starting from the first graph. After a little time had gone by, she asked them to scan through the equations to see if there were ones they could match up quickly with a graph. She hoped that they would recognize $y = 4$, $x = 2$, and maybe $y = x$, as easy to spot. This wasn't necessarily the case. But, it gave an opportunity to discuss again these special graphs.

The instructor then reminded participants that they had worked with slope and y-intercept, helped them review that a bit, and asked if it could be helpful here. Once participants were thinking that way, slope and y-intercept proved to be an effective way for participants to do the matching.

3. In two different classes, the instructor used a total of 5 of the experiments from Algebra Experiments I, using the instructions and worksheets given in the book. See Comments below.

Performance Assessment:

- Review activities in the text(s) were one way the instructor assessed performance.
- She also monitored the work participants did in class, paying attention not only to what they were able to do, but to the questions they asked, the comments they voiced, and mistakes they made.
- Both individual written responses and class discussion of the "Interpret the Data" sections of the Algebra Experiments I worksheets gave the instructor much information about how well participants were understanding the process.

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Comments:

- The instructor was constantly looking for ways to help participants see how algebra interacts with the real world, how it might have practical application. Using Algebra Experiments I was part of that effort, and it was a great success.

The first time the instructor used this material, she did it "cold" (doing little or no graphing of equations work before beginning the experiment). She worried some that the material might be "out of the league" of the participants, but, given the hands-on, experimental focus, she decided it was worth the risk. She introduced concepts or skills such as line of best fit, slope, intercept, function, dependent variable, and independent variable as needed when they were actually doing the work. She also tried to keep instruction matter-of-fact and to the point.

It worked. Participants liked doing the experiments. And even though it wasn't real, on-the-job data they could use, they could see how the process worked.

In the other class, the instructor did have participants work with graphing equations and learning about slope, intercept, etc. before doing the experiments. It seemed somewhat helpful for them to have this background, but this approach was not significantly better than the first approach.

The instructor would have liked to have the time and opportunity to have participants collect some actual on-the-job data, graph it, write an equation, interpret it, etc. That did not happen in either of these classes.

Activity XII: Concluding Procedures

1. Complete grant paperwork
2. Give Algebra / Geometry Post test to participants
3. Provide feedback to participants, and get feedback from participants on class.
4. Complete Individualized Learning Plans (Part I was completed at the beginning of class)

Resources and Materials Needed:

- Grant paper work
- Post test
- Individualized Learning Plans with Part I completed

Description of Instruction:

1. The class completes the paperwork for the grant
2. The class takes the post-test
3. Completing the Individualized Learning Plans

Performance Assessment Methods and Results:

- Participants were given the opportunity to review the progress between the pre and post test results.
- Participants also had a chance to provide input to the instructor on the quality of the class.

Course Summary

Background:

In the Job Task Analysis Report, company personnel identified math training needs at both basic and advanced levels. While some employees needed work and practice in basic skills such as working with fractions, decimals and proportions, others expressed interest in geometry and algebra especially in reference to hydraulics, electricity and other technical areas. Also, managers often said that employees made mistakes on the job because they didn't recognize whether their answers were reasonable or not. The company specifically requested that math classes include estimation and reasoning skills.

Expectations:

Company:

Some supervisors and employees from the maintenance department expressed interest in algebra / geometry. Several members of the Site Advisory Team promoted the class to employees in their work areas, and several SAT members participated in the class as well.

Learner

Most employees who showed interest in participating in the class wanted the opportunity to "re-learn" some math skills and exercise some thinking skills that they had not used for some time. They realized that if they were more proficient in using algebra they could use it in an easier and faster way to work through problems.

Instructor:

The instructor had taught several math classes at this worksite already. From talking with participants in her other classes and having a general knowledge of the company, she recognized some applications of algebra and geometry. She expected that participants in this class would bring work-related problems to class.

Evaluations:

Company:

In response to participants' interest, the Site Advisory Team extended the Algebra / Geometry class beyond the original class sessions. Also, some participants from the Basic Math training expressed interest in a pre-algebra class. The pre-algebra class was later scheduled as a result of those interests.

Learner:

Participants commented on how they improved their skills and expanded their knowledge of math concepts. Post test scores showed significant gains. Participants commented on learning systematic ways to organize their thinking. For many of them, their confidence in estimating, in using math on-the-job, and in asking questions changed positively during the

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time of the training. One person learned much of the material but showed less confidence about using this material on tests. Despite this difficulty with test performance, he successfully applied ratio and proportions to work situations and overall felt that he had gained significant work-related skills.

Instructor

By the time this class was offered, the instructor knew the workplace well enough to recognize, accumulate, and organize workplace applications for decimals, proportions, percents, and geometry. She realized that in order to incorporate the application of algebraic concepts to the workplace, two requirements would have to be met: 1) the instructor would need to become even more familiar with company operations because workplace applications of algebra were not as obvious; 2) participants would need to understand the algebra content thoroughly enough to recognize work situations where algebra could be applied. Time constraints prevented addressing these requirements. As a consequence, the course concentrated on algebra skills rather than applied algebra.

HANDOUTS

Algebra/Geometry Outline

Review and Background

- Number concepts and patterns

- Estimation

- Decimals

- Fractions

Ratio and Proportion

Percents

Geometry

- Perimeter, area, volume

- Pythagorean theorem

- Angles, lines, triangles, etc.

Algebra

- Order of operations

- Using variables in phrases and equations

- Solving equations

- Formulas

Algebra/Geometry Assignment

In order for me to make this class as helpful to all of you as possible, I need to know as much as possible about your job. I am especially interested in problems you have to solve, decisions you have to make, math that you use frequently, math that comes up occasionally that you feel uncertain about how to do, situations in which you use reasoning, estimation, etc.

With that in mind, I'd like you to do two things for me. First, I want you to outline for me what a typical day involves for you: what are you likely to be doing on an hour by hour basis? This can be brief, but try to include the most significant and the most typical stuff. Also, if part of your day requires filling out reports or other paperwork, please bring me a copy of any forms you use or, if there is no form, try to explain what information you must report and whether any of it seems math-related.

The second thing I'd like you to do is write up for me a math problem that might arise in the course of your job. It can be as simple or as complex as you want, though it will be most helpful to all of us if it if you try to either present a situation that is either typical of what you do or a situation that you have trouble figuring out (or both). Try to describe the situation and the information you have to work with as clearly as possible, then state the question to be answered. For example, a question I've used from production is:

You are operating line #3 at 1000 pounds per hour and standard rate for that line is 1250 pounds per hour. At what percent of standard are you operating? If the extruder rpm is 1700 now, what theoretical rpm would it take to get to standard rate?

Inverse Variation

Quantities are said to *vary inversely* when an increase in one produces a decrease in the other (or a decrease in one produces an increase in the other). For instance, the number of days required to complete a job varies inversely as the number of workers working on it (if they all work at the same rate). Using d for time in days, w for the corresponding number of workers, D for the new time in days, and W for that corresponding number of workers, the resulting proportions could look like:

$$\frac{d}{D} = \frac{W}{w}$$

$$\frac{D}{d} = \frac{w}{W}$$

Set up appropriate proportions for the following problem:

If 8 men do a job in 9 days, how long do 24 men take to do the same job?

(Optional: Note that here we can create a unit that we might call worker-days, which is the product of the number of days and the corresponding number of workers (in this case, that product is 72, so it takes 72 worker-days to complete this job. Algebraically, $dw=k$.)

1. At 40 mph, how fast is a journey which takes 6 hours at 30 mph?
2. The volume of a gas is 40 cubic feet under 3 pounds pressure. What is its volume at the same temperature when the pressure is 8 pounds?
3. If the current is 18 amperes when the resistance is 5 ohms, what is the current when the resistance is 3 ohms?
4. The winner of a race ran the distance in 45 seconds at an average speed of 9.6 meters/second. The runner who came in last finished in 48 seconds. What was the last runner's average speed?
5. A 15-centimeter pulley runs at 300 rpm. How fast does the 5-centimeter pulley it drives revolve, if the number of rpms varies inversely as the diameter?
6. The amount of current, I , flowing through a circuit is inversely proportional to the amount of resistance, R , of the circuit. In a circuit with a resistance of 18 ohms, there is a current of 0.25 A. If the resistance in a circuit were .65, what amount of current should be flowing through that circuit?
7. A gear with 18 teeth makes 100 rpm and meshes with a gear having 12 teeth. Find the speed of the second gear if rpm varies inversely with number of teeth.

Review of Direct and Inverse Variation

Here are some examples of problems involving both direct and inverse variation. To solve, first decide whether one variable increases as the other increases OR if one variable increase as the other decreases. Then set up a proportion and solve.

1. If 3 gallons of paint cover a surface containing 825 square feet, how many gallons at the same rate will be needed to paint a surface containing 1925 square feet?
2. A building casts a shadow of 28 feet when a boy, 5 feet tall, casts a shadow of 2 feet. Find the height of the building.
3. A driver gear has 40 teeth and makes 20 rpm. If the driven gear has 16 teeth, how many rpm will it make?
4. A driver gear has 36 teeth and makes 150 rpm. How many teeth must the driven gear have if it is to make 225 rpm?
5. If the actual distance of 29 feet is represented by the scale distance of $3\frac{5}{8}$ " on a floor plan, what scale distance will represent the actual distance of 34 feet?
6. A beam bends 1.6 cm. with a mass of 32 kg. on it. If the amount of bending is directly proportional to the mass, find the amount of bending caused by a mass of 62 kg.
7. A room is to be partitioned into a row of small compartments. If each compartment is 1.8 meter wide, there will be room for 16 compartments. If only 15 compartments are constructed, how wide will each compartment be?
8. If a plane goes 600 miles in $1\frac{3}{4}$ hours, how far will it go in 7 hours at the same rate?
9. When an electric current is 24 A, the electromotive force is 216 V. Find the amount of current when the force is 405V.
10. If 12 feet of a particular wire have a resistance of .00324 ohms, what is the resistance of 350 feet of this wire?
11. If a driver pulley has a diameter of 15" and makes 600 rpm, how many rpm does a driven pulley 30" in diameter make?
12. A driven pulley is 6" in diameter and needs to make 1250 rpm. What size pulley will be needed to drive it if the driver pulley makes 750 rpm?
13. Rod A has 80 equal divisions, and rod B has 100, although they are the same length. If lengths which equal 60 divisions of rod B are cut from each, how many divisions are cut from rod A.

NOTATION WORKSHEET 3a
USING EXPONENTS

Directions: Fill in the table below.

Exponent Form	In English Words	Repeated Multiplication	The Value
10^4	ten to the fourth power	$10 \times 10 \times 10 \times 10$	10,000
5^2	five squared	(5) (5)	25
2^3			
		(4) (4) (4)	
		(3) (3) (3) (3)	
	eight to the third power		
	ten cubed		
0^3			
	the second power of 12		
0^6			
2^4			
16^2			
4^1			
1^4			

NOTATION WORKSHEET 4a

SQUARE ROOTS AND SQUARES

Directions: Fill in the empty boxes with numbers.

Example :

$$\square \times \square = 36 \text{ (same number in each box)}$$

becomes $6 \times 6 = 36$

$$\square \times \square = 49$$

$$\square^2 = 100$$

$$\square \times \square = 121$$

$$\square^2 = 25$$

$$\square^2 = 4$$

$$\square^2 = 36$$

$$\square \times \square = 9$$

$$\square \times \square = 1$$

$$\sqrt{16} = \square$$

$$\sqrt{64} = \square$$

$$\sqrt{81} = \square$$

$$\sqrt{144} = \square$$

$$\sqrt{100} = \square$$

$$\sqrt{900} = \square$$

$$\sqrt{25} = \square$$

$$\sqrt{8,100} = \square$$

Scientific Notation

Some numbers are so large or so small that they are difficult to read or to write. For example, consider the following:

diameter of the solar system: 118,000,000,000 km

diameter of a silver atom: 0.00000000000025

Scientific notation is based on positive and negative powers of 10. You are familiar with the positive powers of 10. The negative powers are defined below.

Positive Powers of 10

$$10^1 = 10$$

$$10^2 = 100$$

$$10^3 = 1000$$

$$10^4 = 10,000$$

and so on

Negative Powers of 10

$$10^{-1} = \frac{1}{10^1} = \frac{1}{10} = 0.1$$

$$10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$$

$$10^{-3} = \frac{1}{10^3} = \frac{1}{1000} = 0.001$$

$$10^{-4} = \frac{1}{10^4} = \frac{1}{10,000} = 0.0001$$

and so on

Positive and negative powers of 10 are used in our decimal number system, as well as in scientific notation, since every digit of a number can be associated with an integral power of 10. For convenience, the zero power of 10 is defined as 1. The following numbers are written in **expanded notation**.

$$3572 = 3 \cdot 10^3 + 5 \cdot 10^2 + 7 \cdot 10^1 + 2 \cdot 10^0$$

$$0.8329 = 8 \cdot 10^{-1} + 3 \cdot 10^{-2} + 2 \cdot 10^{-3} + 9 \cdot 10^{-4}$$

$$31.407 = 3 \cdot 10^1 + 1 \cdot 10^0 + 4 \cdot 10^{-1} + 0 \cdot 10^{-2} + 7 \cdot 10^{-3}$$

Write the following numbers in expanded notation.

1. 23,891 _____

2. 75.102 _____

3. 0.0608 _____

The metric system of measurement is also based on powers of 10. Conversions from one metric unit to another simply involve multiplying by a power of 10. **For example:**

To convert from meters to kilometers, multiply by 10^{-3}

To convert from kilometers to meters, multiply by 10^3

A number is written in **scientific notation** when it is expressed as a product of a number greater than or equal to 1 but less than 10, and an integral power of 10. That is, scientific notation translates a number to the form: ($1 \leq \text{number} < 10$ times 10^n). See example below.

$$118,000,000,000 = 1.18 \times 10^{11} \quad \text{decimal point moved 11 places}$$

$$4,709,000,000 = 4.709 \times 10^9 \quad \text{decimal point moved 9 places}$$

$$0.000368 = 3.68 \times 10^{-4} \quad \text{decimal point moved 4 places}$$

$$0.00000000000059 = 5.9 \times 10^{-13} \quad \text{decimal point moved 13 places}$$

Express each of the following numbers in scientific notation.

1. 38,500 _____ 2. 4,070,000 _____

3. 18,050,000,000 _____ 4. 0.003 _____

5. 0.000028 _____ 6. 0.0000000902 _____

Note that it can be easier to compare large or small numbers if they are expressed in scientific notation.

$$\text{Distance from sun to Pluto} = 5.9 \times 10^9 = 59 \times 10^8$$

$$\text{Distance from sun to Mercury} = 6 \times 10^8 = 6 \times 10^8$$

$$\text{Ratio} = 59:6$$

Numbers written in scientific notation can be multiplied and divided easily by using the rules of exponents. **For example:**

$$(3.2 \times 10^7)(2.0 \times 10^4) = (3.2 \times 2.0)(10^7 \times 10^4) = 6.4 \times 10^{11}$$

(Add exponents when multiplying--when the bases are the same.)

$$\frac{3.2 \times 10^7}{2.0 \times 10^3} = \frac{3.2}{2.0} \times \frac{10^7}{10^3} = 1.6 \times 10^4$$

(Subtract exponents when dividing--when the bases are the same.)

$$(2.5 \times 10^3)(6.0 \times 10^2) = (2.5 \times 6.0)(10^3 \times 10^2) = 15 \times 10^5 = 1.5 \times 10^6$$

Express each of the following in scientific notation.

1. Light travels at a speed of 186,000 miles per hour. _____
2. Our Milky Way is about
600,000,000,000,000,000 miles in diameter. _____
3. An inch is about .000016 mile. _____
4. The sun at any second develops
500,000,000,000,000,000,000 horsepower. _____
5. The diameter of a large molecule is .00000017 centimeter. _____
6. A starfish lays an average of 2,520,000 eggs annually. _____
7. The charge of an electron is .00000000048 electrostatic units. _____
8. The length of the largest virus (parrot fever)
is .000025 centimeter. _____

On the lines below, write scientific notation for 2 large and 2 small numbers that might occur in your work. Be sure to label these numbers so I know what they mean.

9. _____
10. _____
11. _____
12. _____

1. The mass of the sun is about 2.0×10^{30} kg. _____
2. The frequency of an AM
radio wave is 1.4×10^6 cycles per second. _____
3. The wavelength of ultraviolet light is 1.36×10^{-6} cm. _____
4. The wavelength of gamma rays is 3.0×10^{-10} cm. _____
5. The diameter of the nucleus
of a hydrogen atom is 5.0×10^{-17} cm. _____
6. The mass of an atom of helium is 6.65×10^{-24} g. _____

Simplify each expression to a power of 10.

1. $10^3 \cdot 10^4 \cdot 10^8$ _____ 2. $\frac{10^5 \cdot 10^4}{10^2}$ _____ 3. $\frac{10^8}{10^6 \cdot 10^6}$ _____

4. $\frac{10^5}{10^{-2}}$ _____ 5. $\frac{10^{-4}}{10^{-6}}$ _____ 6. $\frac{10^4 \cdot 10^{-2}}{10^{-1}}$ _____

Perform the indicated operations.

1. $(2 \times 10^5)(4 \times 10^6)$ _____ 2. $\frac{6 \times 10^7}{2 \times 10^2}$ _____ 3. $(4 \times 10^6)(3 \times 10^2)$ _____

4. $\frac{8 \times 10^6}{2 \times 10^{-3}}$ _____ 5. $\frac{9 \times 10^{-5}}{3 \times 10^{-3}}$ _____ 6. $\frac{(4 \times 10^4)(9 \times 10^{-2})}{2 \times 10^5}$ _____

7. $\frac{6 \times 10^8}{3 \times 10^2}$ _____ 8. $(1.5 \times 10^7)(3 \times 10^4)$ _____ 9. $(8 \times 10^4)(2 \times 10^4)$ _____

10. $\frac{(5 \times 10^7)(9 \times 10^{-3})}{3 \times 10^{-2}}$ _____ 11. $\frac{(2 \times 10^5)^2}{(8 \times 10^{-3})(5 \times 10^{12})}$ _____ 12. $\frac{(4 \times 10^{-5})^3}{(2 \times 10^{-7})^2}$ _____

More Practice with Scientific Notation

Solve the following problems, then check your answers using scientific notation on your calculator. Please show your work.

1. $\frac{6.9 \times 10^{-6}}{2.3 \times 10^{-2}}$

2. $(5.83 \times 10^{-9}) \times (1.39 \times 10^{-8})$

3. $\frac{1.82 \times 10^5}{9.1 \times 10^{17}}$

4. $(9.72 \times 10^8) \times (4.8 \times 10^{-20})$

5. $8.3 \times 10^3 + 2.7 \times 10^4 + 3.1 \times 10^4 - 6.7 \times 10^3 - 8.1 \times 10^4$

Use the scientific notation function on your calculator to solve the following:

6. $(3.4 \times 10^5) (2.4 \times 10^3)$

7. $(6.4 \times 10^{-3}) (7.7 \times 10^1)$

8. $(2.653 \times 10^3) (2.52 \times 10^{-1})$

9. $(3.67 \times 10^{-3}) (3.587 \times 10^{-2})$

10. $\frac{(5.26 \times 10^{-2}) (6.3 \times 10^{-3})}{4.06 \times 10^{-4}}$

11. $\frac{(3.200 \times 10^3) (8.72 \times 10^4)}{5.66 \times 10^5}$

TIME TO EXPLAIN YOURSELF

Write an explanation for each of the following.

1. Explain how you would draw a representation of $3x^2$.
2. Explain how you would draw a representation of $(3x)^2$.
3. Explain the difference between $3x^2$ and $(3x)^2$.
4. Explain how to calculate 7^5 on your calculator.
5. Explain how to calculate 9^3 on your calculator.
6. Explain how to convert a number to scientific notation using your own brain.
7. Explain how to convert a number to scientific notation using your calculator.

How Many?

Answer the following questions.



How Many . . .

1. degrees in a circle?	360°
2. lines of symmetry in a square?	
3. diagonals in a square?	
4. diameters in a circle?	
5. vertices in a cube?	
6. fractions between $\frac{1}{2}$ and $\frac{1}{3}$?	
7. whole-number factors of 36?	
8. two-digit multiples of 13?	
9. palindromes between 100 and 200?	
10. square feet in a square yard?	
11. numbers have an absolute value equal to 5?	
12. quarter-ounces in a pound?	



. . . AND THINK

List another problem similar to the ones presented on this page. Provide the related question and the numerical answer to the "How Many?" question.

Review of Geometrical Terms and Figures

Point

Ray

Line

Line segment

Vertical line

Horizontal line

Parallel lines

Perpendicular lines

Intersect

Angle

Vertex

Rotation

Degree

Right Angle

Acute Angle

Straight Angle

Obtuse Angle

Complementary Angles

Supplementary Angles

Adjacent Angles

Vertical Angles

Corresponding Angles

Polygon

Octagon

Hexagon

Pentagon

Quadrilateral

Trapezoid

Parallelogram

Rhombus

Rectangle

Square

Circumference

Radius

Diameter

Triangle

Equilateral Triangle

Isosceles Triangle

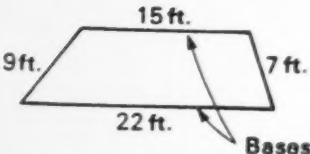
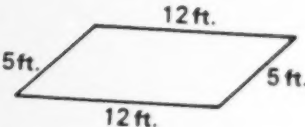
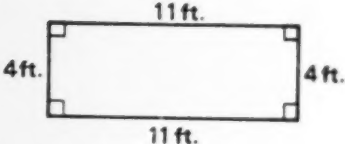
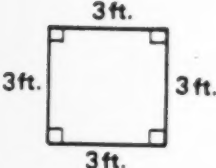
Right Triangle

Symmetry

Reflection Symmetry

Plane Geometric Figures

A polygon is a plane (flat) figure formed by three or more straight lines. Several types of polygons are given special names. Some of those are shown below.

<u>Name</u>	<u>Example</u>	
Triangle		A <u>triangle</u> is a polygon formed by three straight lines.
quadrilateral		A <u>quadrilateral</u> is a polygon formed by 4 straight lines.
Trapezoid		A <u>trapezoid</u> is a special quadrilateral with one pair of opposite sides parallel. These parallel sides are called the bases of the trapezoid.
Parallelogram		A <u>parallelogram</u> is a special quadrilateral with both pairs of opposite sides parallel. The opposite sides of a parallelogram are also equal. A parallelogram is rather like a trapezoid with both pairs of sides parallel. If all sides of a parallelogram are equal, the figure is called a <u>rhombus</u> .
Rectangle		A <u>rectangle</u> is a special parallelogram with four right angles.
Square		A <u>square</u> is a special rectangle with all its sides equal. (You could also call a square a special rhombus with four right angles.)

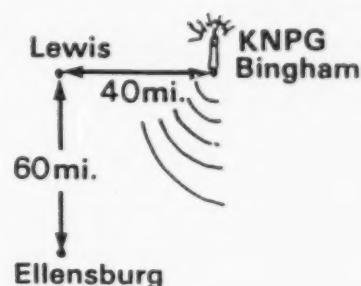
The perimeter of a figure is the distance around that figure. Perimeter is measured in linear units such as feet, meters, inches, miles, and kilometers.

The area of a figure is the amount of space enclosed in that figure. Area is measured by how many square pieces (square units such as square feet, square

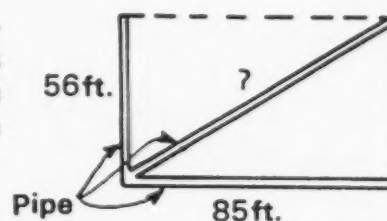
Practice with Pythagoras

1. How long is a path running diagonally across a rectangular lot 405 feet long and 215 feet wide?
2. If the bottom of a 50' ladder is placed 14 feet from a wall, how high up on the wall does it reach?
3. What is the distance from the pitcher's box to second base if the distance from the pitcher's box to home plate is 60 feet and the distance between bases is 90 feet?
4. In the installation of an antenna 16 feet high, how long must a wire be to reach from the top of the antenna to a point on the roof level with the antenna but 12 feet from its base?
5. A boat left port, sailing 28 miles due west and 45 miles due south. How far away from the port was the boat?

6. Radio station KNPG in Bingham has a broadcast range of 70 miles. Refer to the drawing at the right and determine if Ellensburg is within the broadcast range of KNPG.



7. Eva Lewis wants to put an underground sprinkler system in her back yard. A drawing of the system is shown at the right. About how many feet of water pipe will Eva need? (*Hint: The unmeasured length of pipe is equal to the hypotenuse of a right triangle.*)



8. Hank is laying out the area for a rectangular barn with dimensions of 24' x 32'. What should the diagonal measurement be?
9. A forest ranger at Oak Ridge Lookout spotted a fire 24 miles west of his location. If the town of Olene is 38 miles due south of him, how far is Olene from the fire?

UNIT CONVERSION or DIMENSIONAL ANALYSIS

by

ROSE STEINER AND MARCIA PAPPAS

Units are names or labels on numbers. For instance, if we have 10 miles, "10" is the number and "miles" is the unit. If we have 24 hours, "24" is the number and "hours" is the unit. "Unit conversion" simply means changing a value from one unit into another, arriving at a corresponding new number. Ratios and proportions are often used to solve this type of problem. For example, if we are to change 2 feet into inches, we use the ratio of "12 inches : 1 foot" to determine that there are 24 inches in 2 feet. This is done formally, with a proportion problem, or informally, just by thinking " $12 \times 2 = 24$ ". This is where a lot of students run into trouble. They know that 12 inches is equal to 1 foot, but they are unsure whether to multiply or divide, so they **guess**. Some students **think** about whether they are changing to a smaller unit (hence, they want a larger number so they know to multiply) or to a larger unit (hence, they will get less of them, so they know to divide). There is a method of doing unit conversion problems that eliminates the need to **"think"** or **"guess"** whether to multiply or divide. That method is **DIMENSIONAL ANALYSIS**.

Dimensional analysis is just another approach to converting values from one unit to another, so-called because it uses a method of "analyzing the dimensions" of the values in addition to working with the numbers themselves. Dimensional analysis was probably first taught in the chemistry classroom in the mid-60's. Prior to its use, only one chemistry conversion at a time could be completed with the ratio-proportion method. Using this new method the complete problem was set up before any of the math was computed. Sometimes the setup of the problem covered a whole sheet of paper. At first, not all chemistry professors were aware of this technique, so students using dimensional analysis were able to finish a problem before the teacher who was using the old ratio-proportion technique.

DIMENSIONAL ANALYSIS USES 3 BASIC CONCEPTS:

1. THE NUMBER ONE (1) IS THE MULTIPLICATIVE IDENTITY.

That is, you can multiply or divide any quantity by one (1), and the result will still be equal to the original quantity.

$$\boxed{6 \times 1 = 6}$$

2. THE NUMERAL ONE (1) CAN BE WRITTEN IN MANY DIFFERENT WAYS WHEN WRITTEN IN FRACTION FORM.

$$\boxed{= \frac{2}{2} = \frac{3}{3} = \frac{7}{7} = \frac{41}{41} = \frac{15}{15} = \text{etc.}}$$

As long as the numerator of the fraction equals the denominator of the fraction, the fraction is equal to one (1). Any definition involving units can be made into a conversion factor equal to 1. For example, since you know, by definition, that 12 inches equals 1 foot, you may say:

$$\boxed{= \frac{12 \text{ inches}}{1 \text{ foot}}} \quad \text{or} \quad \boxed{= \frac{1 \text{ ft}}{12 \text{ in}}}$$

There are an infinite number of **unit definitions** (conversion factors), each equal to one (1).

$$\boxed{= \frac{60 \text{ minutes}}{1 \text{ hour}} = \frac{2 \text{ lb}}{32 \text{ oz}} = \frac{1 \text{ meter}}{100 \text{ centimeters}} = \frac{365 \text{ days}}{1 \text{ year}} = \frac{3 \text{ ft}}{36 \text{ in}}}$$

3. FRACTIONS MAY BE REDUCED OR "CANCELED" IF THE NUMERATOR AND DENOMINATOR HAVE A COMMON FACTOR.

For example, in the problem below, you cancel the common factor of four (4), and the common factor of five (5).

$$\begin{array}{c} 1 \quad 1 \\ \frac{4}{5} \times \frac{5}{16} = \frac{1}{4} \\ 1 \quad 4 \end{array}$$

With dimensional analysis, not only do numbers cancel, but units that are exactly the same can also cancel. For example:

$$\frac{3 \text{ hr}}{1} \times \frac{60 \text{ min}}{1 \text{ hr}} = 180 \text{ min}$$

Dimensional analysis is an easy problem solving technique that uses simple fraction multiplication to solve complex conversion problems.

The steps used to set up a dimensional analysis problem:

1. Determine what unit is given and what unit you are seeking in the answer. This is often the most challenging step in this technique.
2. Write down what is known about the units involved in the problem.
3. Start at the left margin of your paper with the given unit in the **numerator**--when in doubt you put the original unit on top.
4. To get rid of the original unit, place that same unit in the next fraction's **denominator**, where it will be **ready to cancel** later.
5. Search for a unit definition that will get you **closer** to the unit you want in the final answer. This becomes the new fraction's numerator. The resulting fraction or "unit definition" **must be** equivalent to **one (1)**.
6. Continue this procedure until the unit has changed into the actual unit you are seeking in your answer. When you arrive at the final unit, **circle it**.
7. **Cancel the unit names** after the unit definitions have all been verified.
8. The last step is to **cancel any common factors in the numbers**; then multiply and divide the remaining numbers, completing the math.
9. The answer is a new number with its corresponding units.

UNIT CONVERSION/DIMENSIONAL ANALYSIS

Selected Examples

Example 1: 1 year = _____ seconds

$$\frac{1 \text{ year}}{1} \times \frac{365 \text{ days}}{1 \text{ year}} \times \frac{24 \text{ hours}}{1 \text{ day}} \times \frac{60 \text{ min}}{1 \text{ hour}} \times \frac{60 \text{ sec}}{1 \text{ min}} = 31,536,000 \text{ seconds}$$

Example 2: 55 miles per hour = _____ feet per second

$$\frac{55 \text{ miles}}{1 \text{ hour}} \times \frac{5280 \text{ feet}}{1 \text{ mile}} \times \frac{1 \text{ hour}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ sec}} = \frac{242 \text{ feet}}{3 \text{ sec}} \text{ or } 80 \frac{2}{3} \text{ feet per second}$$

Example 3:

20 cubic yards = _____ cubic inches

$$\frac{20 \text{ yd yd yd}}{1} \times \frac{36 \text{ in}}{1 \text{ yd}} \times \frac{36 \text{ in}}{1 \text{ yd}} \times \frac{36 \text{ in}}{1 \text{ yd}} = 933,120 \text{ cu in}$$

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Practice with Unit Conversion/Dimensional Analysis

Use the dimensional analysis to set up the following unit conversion problems. Then compute.

1. 43,000 seconds = _____ day
2. 40 mph = _____ ft/ second
3. 2.75 sq. yd. = _____ sq. in.
4. 60 mph = _____ yard/second
5. A patient is to be given 250 mL of intravenous medication in 12 hours, how many microdrops is this per minute if 60 microdrops is 1 mL?
6. In moving to Ohio, Sue drove 10 hours per day (averaging 55 mph) in a U-Haul truck that averaged 5 miles per gallon of gas. If the average cost of gas was \$1.25, how many dollars did she spend on gas per day of driving?
7. Find the approximately annual wage of a person who works 8 hours a day, 5 days a week and earns \$9.75 per hour.

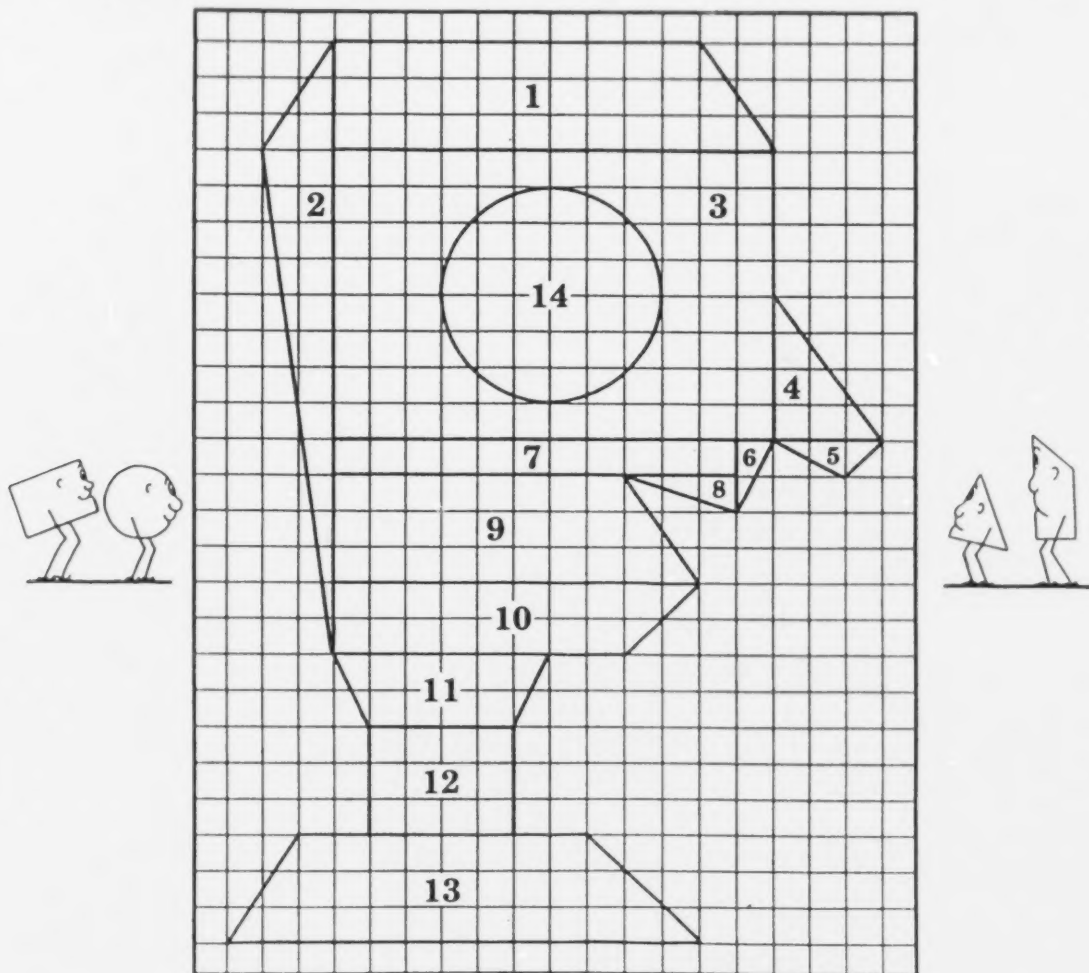
8. Suppose a guard at a hazardous waste site is required to make 2 trips around the site each hour. The site is 4000 feet around. What is the total distance, in miles, walked during an 8-hour shift?
9. Suppose an avalanche travels at 200 mph, how many yards will it travel in 10.4 seconds?
10. How many seconds would it take an avalanche at 200 miles per hour to travel the length of a city block (500 ft.)?
11. A lawn mower cuts 10 feet in 3 seconds. If the oil in the lawnmower needs to be changed every 25 hours (and the lawnmower is always shut off if it is not mowing), how many miles will it travel between oil changes?
12. If a semi driver pays \$.07 per gallon in road taxes, gets 5 miles per gallon, and averages 50 mph, how much does he pay in road taxes in a year of driving 9 hours a day, 5 days a week?

UNIT CONVERSION IN THE WORKPLACE

13. Silo 1 contains regrind that weighs 42 pound per cubic foot and each foot of height contains 140 cubic feet. If the silo has 20 feet of height empty, how long will it take to fill the silo if you are grinding into it at 1100 pounds per hour?
14. Material costs are a 30 cents per pound. You are running a line of 8" 3034 and the target weight per foot is 3.31 pounds. You are at an 8% overweight and will produce 2800 feet of pipe in the 12-hour shift. What is the total material cost of the pipe going to be?

Finding the Area of "Old Shadowface"

Find the total area of the outline of "Old Shadowface" as diagrammed below. Don't forget to subtract the hole in the head. The fourteen figures that make up the face include triangles, rectangles, trapezoids, and a circle. Each unit of grid represents 3 centimeters; use $\pi = 3.14$.



Areas

1. _____	5. _____	9. _____	13. _____
2. _____	6. _____	10. _____	14. _____
3. _____	7. _____	11. _____	Total Area
4. _____	8. _____	12. _____	_____

REVIEW

1. 3% of 600 is _____
2. What percent of 120 is 18?
3. 8 is 5% of what number?
4. The cost of a restaurant meal, including a 7% tax, was \$16.05. What was the price of the meal, not including tax?
5. An \$85 pair of running shoes is marked down 15%. What is the sale price?
6. Paul was given a 32% discount on a new stereo. If this saved him \$86, what was the original price of the stereo?
7. What is the percent of discount if a shop reduces the price of an item from \$80 to \$68?
8. A gear with 36 teeth revolves at 800 rpm and meshes with a gear with 24 teeth. Find the speed of the second gear.
9. Two thermometers are exactly the same except that thermometer F is marked off into 180 equal units and thermometer C is marked off into 100 equal units. A length equal to 66.6 units on thermometer F is equal to a length of how many units on thermometer C?
10. How much would you have to invest at 8% to earn as much interest as \$1250 invested at 12%?
11. If the radius of a circle is increased by 100%, how is the area increased?
12. A recipe for pizza dough calls for a 10" x 14" pan. If you want to make a circular pizza approximately the same thickness, what should be the diameter of the pan?
13. If 75 is divided into three parts in the proportion 2 to 3 to 5, the smallest part is _____
14. If 6 men can do a job in 7 days, at the same rate how long would it take 18 men to do the same job?
15. A 2-foot pipe is to be cut into 2 parts having the ratio 2:1. What is the length of each part?
16. The time needed to fill a tank varies inversely as the square of the radius of the hose. If a hose of radius 3.5 cm takes 8 minutes to fill a tank, how long will it take using a hose of radius 2 cm?

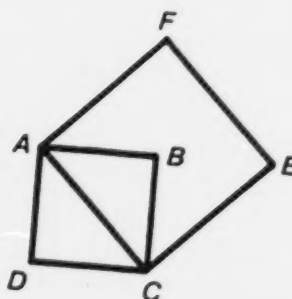
17. How many cubic yards of topsoil will it take to cover a garden area 30' by 60' with a 3" layer of topsoil?

18. Pipe with inside diameter of 21" is used to drain water after a certain manufacturing process. How much gallons of water will 100' of this pipe hold? (1 cu. ft. = 7.5 gal.)

19. The angles of a triangle are in the ratio of 3:4:5. How many degrees are there in each angle?

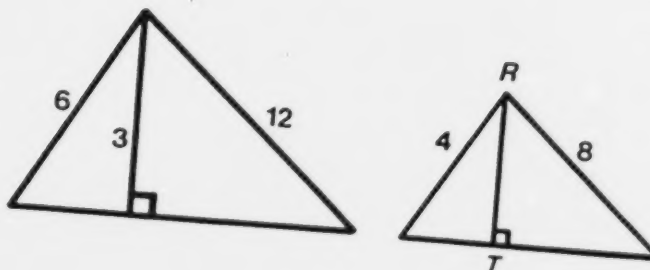
20.

$ACEF$ is a square. $ABCD$ is a square with each side 1 unit long. What is the ratio of the area of $ACEF$ to the area of $ABCD$?



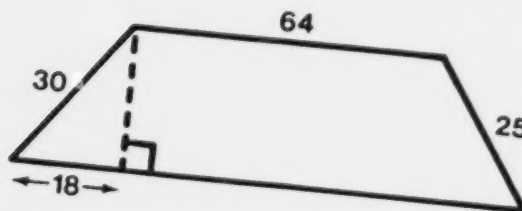
21.

The two triangles are similar. How long is $\overline{RT}$?



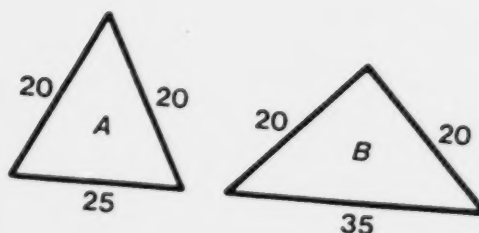
22.

Find the perimeter.



23.

Which triangle has the greater area?



Formulas and Definitions

Pythagorean Theorem: $a^2 + b^2 = c^2$ or

In any right triangle, adding the squares of the sides forming the right angle equals the square of the hypotenuse (side opposite right angle).

Perimeter is the distance around some 2-dimensional space.

To find **perimeter** of a straight-sided figure, add all sides.

Circumference is the distance around a circle.

$$\text{Circumference} = \pi \times \text{diameter}$$

Area is the space enclosed by a 2-dimensional shape and measured by the number of 1-unit squares that will fit in that space.

$$\text{Area of a rectangle (square)} = \text{length} \times \text{width}$$

$$\text{Area of a triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\text{Area of a parallelogram} = \text{base} \times \text{height}$$

$$\text{Area of a trapezoid} = \frac{1}{2} \times \text{sum of the two bases} \times \text{height}$$

$$\text{Area of a circle} = \pi \times \text{radius}^2 \text{ or } \text{diameter}^2 \times .785$$

Vocabulary:

height: the perpendicular distance from the highest point to the base

diameter: distance across a circle through the center (twice the radius)

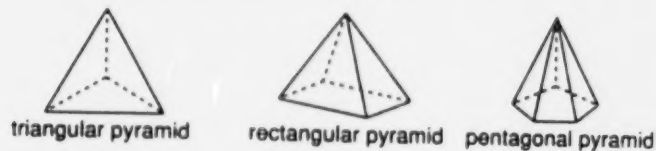
radius: distance from the center of a circle to the edge (half the diameter)

π : ratio of the circumference of a circle to its diameter. Approximately 3.14 or 3.1416 or $\frac{22}{7}$.

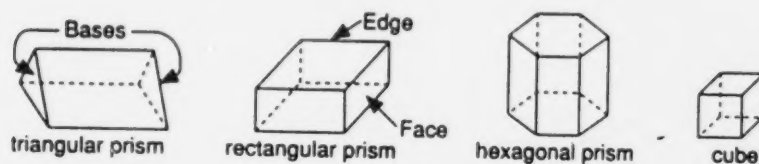
VOLUME is the space enclosed by a 3-dimensional shape and measured by the number of 1-unit cubes that will fit in the space.

Here are some common 3-dimensional (solid) figures.

Pyramids



Prisms



**Other
Three-dimensional
Figures**



Volume of a cylinder or prism = area of the base $\times$ height

Volume of a pyramid or a cone = $\frac{1}{3} \times$ area of the base $\times$ height

Volume of a sphere = $\frac{4}{3} \times \pi \times \text{radius}^3$

SURFACE AREA is the total area of all the surfaces of a 3-dimensional figure.

Surface area of a cone = $\pi \times \text{radius}^2 + \pi \times \text{radius} \times \text{length of side of cone}$

Surface area of a sphere = $4 \times \pi \times \text{radius}^2$

VOLUME

WORKSHEET 7

On the GED, a page of formulas will include the following three which deal with volume:

VOLUME (V) of a:

cube

$$V = s^3; \text{ where } s = \text{side}$$

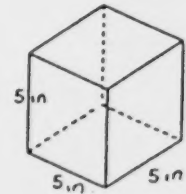
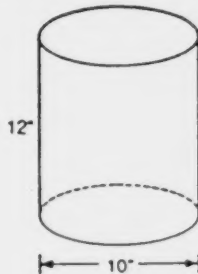
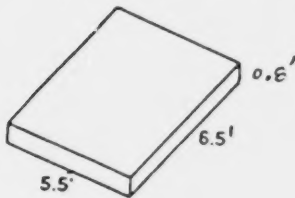
rectangular container

$$V = lwh; \text{ where } l = \text{length, } w = \text{width, } h = \text{height}$$

cylinder

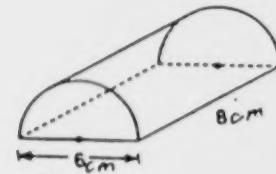
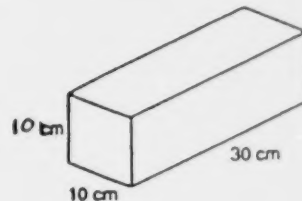
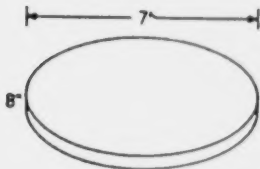
$$V = \pi r^2 h; \text{ where } \pi = 3.14, r = \text{radius, } h = \text{height}$$

For each problem below, select the appropriate formula, then "set up" the particular problem, using the information given.

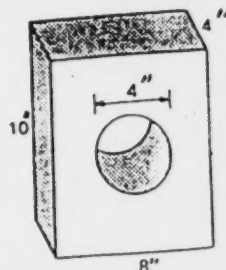


$$V = lwh$$

$$V = (5.5')(6.5')(0.8')$$



challenge



challenge!
combine two formulas!

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Using Algebra to Describe Patterns

Copy each statement onto an index card and shuffle the cards. Without looking at this sheet, sort the statements into groups of similar characteristics. (Your groups should match the groups shown here.) Then write an algebraic equation on each blank line to describe the pattern in that group.

$$5 \cdot 0 = 0$$

$$0 \cdot 3 = 0$$

$$\left(\frac{3}{4}\right)(0) = 0$$

$$(.2)(0) = 0$$

$$0 \cdot 1 = 0$$

$$9 \cdot 1 = 9$$

$$\frac{7}{8} \cdot 1 = \frac{7}{8}$$

$$(.6)(1) = .6$$

$$(1)(1) = 1$$

$$(-5)1 = -5$$

$$2 \cdot 2 = 2^2$$

$$(-6)(-6) = (-6)^2$$

$$3 \cdot 3 = 3^2$$

$$\left(\frac{5}{6}\right)\left(\frac{5}{6}\right) = \left(\frac{5}{6}\right)^2$$

$$(.9)(.9) = (.9)^2$$

$$4 \cdot 7 = 7 \cdot 4$$

$$\frac{3}{10} \cdot \frac{1}{3} = \frac{1}{3} \cdot \frac{3}{10}$$

$$(.4)(.6) = (.6)(.4)$$

$$(-5)(2) = (2)(-5)$$

$$6 + 0 = 6$$

$$\frac{2}{3} + 0 = \frac{2}{3}$$

$$.25 + 0 = .25$$

$$72 + 0 = 72$$

$$34 - 34 = 0$$

$$9.4 - 9.4 = 0$$

$$\frac{1}{4} - \frac{1}{4} = 0$$

$$5 - 5 = 0$$

$$5 + 4 = 4 + 5$$

$$\frac{2}{5} + \frac{1}{4} = \frac{1}{4} + \frac{2}{5}$$

$$.6 + 1.3 = 1.3 + .6$$

$$3 \cdot \frac{1}{3} = 1$$

$$\frac{7}{8} \cdot \frac{8}{7} = 1$$

$$7 \cdot \frac{1}{7} = 1$$

$$\frac{1}{2} \cdot 2 = 1$$

$$\frac{14}{14} = 1$$

$$\frac{9}{9} = 1$$

$$\frac{3}{3} = 1$$

$$\frac{5}{8} \div \frac{5}{8} = 1$$

Order of Operations
PEMDAS - or - Please Excuse My Dear Aunt Sally

Calculate the value of the following expressions. Remember to follow the order of operations. Do not use your calculator. This is a chance to practice or improve your familiarity with order of operations.

1. $4 + 6 \times 3$

2. $80 \div (22 - 2)$

3. $98 - 6 \div 3$

4. $42 + 9 \times 6$

5. $9 \times 4 - 12 \times 2$

6. $60 \div (5 - 2)$

7. $60 \div 5 - 2$

8. $6 \times (3 + 4) - 9$

9. $20 \times 3 - (6 + 4)$

10. $1 + 2(8 + 4) \div 5$

11. $30 - 7 \times 5 + 18$

12. $14 \times 3 - 25 \div 5$

13. $18 \div 3^2$

14. $(18 \div 3)^2$

15. $2(8 + 9) - 28 \div 7$

16. $9 + 5(7 + 3) - 24 \div 8$

17. $36 - 2(3 + 12 \div 4)$

18. $(20 - 6 \times 2)^2$

Number Tricks

Here's a number trick similar to many you may have experienced:

Think of a number.

Add three to it.

Double.

Take away four.

Halve.

Take away the number you first thought of.

The answer is always the same. What is that answer? _____

Let's model this to see why it works...

Some of the following are tricks. Some are not. Use the blocks to model them (and/or check by trying several numbers to see if you always get the same answer). Then decide which ones are really tricks.

- | | |
|--|--|
| 1. Think of a number.
Add 5.
Multiply by 3.
Subtract 9.
Divide by 3.
Take away "your" number. | 2. Think of a number
Add 1.
Double.
Add 10.
Divide by 4.
Take away "your" number. |
| 3. Think of a number.
Add 3.
Double.
Subtract 2.
Take away "your" number. | 4. Think of a number.
Add 3.
Double.
Add "your" number.
Divide by 3.
Take away "your" number. |

From Modeling to Shorthand (Algebra):

We have been using block and/or pictures to help us imagine the number somebody thought of. Suppose we use the algebra notation of using "x" to stand for an unknown quantity.

Thus: Four times the number somebody thought of, with 6 added, becomes $4x + 6$.

Represent the following using algebra notation...

1. Twice the number thought of. _____
2. Five times the number thought of. _____
3. Three times the number thought of with 2 added. _____
4. Twice the number thought of with 9 added. _____

Finding Endings for Tricks

It does not matter how a trick begins, it can always be finished successfully. Here is the beginning of a trick in words and algebraic notation. First we will model the steps using the blocks. Next, we'll translate that thinking into algebraic notation. Then, we will think of ways we could finish this trick.

Think of a number. _____

Add one. _____

Double. _____

Add 1. _____

Double. _____

Some beginnings of tricks are given below. Find a good end for each of them. There are many different ways in which these tricks could end. If you get a trick that works, your answer is right. Please show the algebraic notation for the whole trick, including your ending for it. (If you are not sure how to work with the algebraic notation, feel free to use the block to model what's happening; then translate to algebraic notation.)

1. Think of a number. _____

Add 3. _____

Double. _____

Subtract 2. _____

2. Think of a number.

Add 5.

Double.

Subtract 6.

3. Think of a number.

Add 1.

Double.

Add 4.

4. Think of a number.

Add 1.

Double.

Add 1.

Multiply by 3.

Add 3.

WORDS INTO SYMBOLS

Write an equation for each sentence.

1. Eight is 7 less than twice the number R .
2. The sum of one-third of the number S and 12 is 23.
3. The product of 12 and the quantity 2 less than the number D is 84.
4. Multiply the sum of twice the number A and 3 by 7, and the product is 126.
5. In S slices of cheese, each containing 115 calories, there are 920 calories.
6. After selling C cans of tomatoes from a 180-can shipment, the store had 38 cans left.

Use the given facts to state an equation that represents the underlined sentence.

7. Yolanda is Y years old. Sara is 2 years younger. The sum of their ages is 34.
8. In April, the O'Briens spent \$150 more for food than for housing. Their food bill was D dollars. For both food and housing they spent \$675 in April.
9. On a trip, the Romeros traveled 640 miles farther by airplane than by automobile. They traveled Q miles by automobile. On the trip, they traveled 900 miles.
10. A house cost 5.5 times as much as the lot it stands on. The lot cost C dollars. The house and lot cost \$105,000.
11. Team A scored one point less than twice team B's score. Team B scored x points. Team A scored 47 points.
12. John is 3 years older than Frank, who is t years old. The sum of their ages is 29.
13. Jill is half as old as her brother who is k years old. The sum of their ages is 36.
14. A bag of potatoes weighing 74 kg weighs half a kilogram more than twice as much as a bag of onions weighing z kg.

Choose a variable to represent the number described by the words in parentheses. Then write an equation that represents the given information.

15. A 17-year-old house is one-fourth as old as a nearby bridge. (Age of the bridge)

16. An astronaut spent 42.8 days in space. This was 3.2 days longer than her previous record. (Number of days in her previous record)

17. Each marble in a bag of 48 marbles is either red or blue. There are 3 more blue marbles than twice the number of red ones. (Number of red marbles)

18. The sum of three numbers is 120. The second of the numbers is eight less than the first, and the third is four more than the first. (First number)

19. The hourly rate for a typist is 75 cents more than the hourly rate for a clerk. In 5 hours, the typist earns as much as the clerk does in 6 hours. (Hourly rate for the clerk)

20. Ten years from now, Sofia will be three times as old as she was eight years ago. (Sofia's age now)

21. If a stone were 5 lb. heavier, its weight would be 1 lb. more than twice what it is. (Weight of the stone)

22. Jill is three times as old as her brother, Jack. Two years ago, the sum of their ages was 12. (Jack's age now)

23. A purse contains \$3.50 in nickels and dimes. There are 16 more nickels than dimes in the purse. (Number of dimes)

24. The length of one side of a triangular lot is 6 feet less than three times the length of the second side. The third side is 8 feet longer than the first side. The perimeter of the lot is 160 feet. (Length of the second side.)

25. Phyl is 2 years older than Maria and Maria is 4 years older than Les. The sum of their ages is 37. (Maria's age)

What's That Number?

Each item below describes a number. Try to identify the number and write it in the space provided to the left of the problem. Time is limited to 20 minutes.

- _____ 1. This number is half as large as the product of 3 and 6.
- _____ 2. Twice this number is 1 less than half of 26.
- _____ 3. This number, when multiplied by 3, is equal to twice the sum of 8 and 7.
- _____ 4. Three times this number is equal to the average of 8 and 16.
- _____ 5. When this number is multiplied by 5, it is equal to twice the sum of 18 and 12 divided by 3.
- _____ 6. Twice the product of 6 and 5 is three times as great as this number.
- _____ 7. When this number is multiplied by 6, the product is 10 greater than the product of 7 and 8.
- _____ 8. The average of this number and the next two consecutive numbers is equal to the difference of 35 and 25.
- _____ 9. When this number is divided by 4, the quotient is $\frac{1}{3}$ as large as the sum of 16 and 11.
- _____ 10. The difference of this number and 9 is twice as large as $\frac{2}{3}$ of 12.
- _____ 11. Five times this number is 4 more than $\frac{3}{4}$ of 48.
- _____ 12. When this number is divided by 4, the quotient is 1 less than $\frac{3}{4}$ of 16.
- _____ 13. The average of this number and the next consecutive odd number is 2 more than $\frac{2}{3}$ of 15.
- _____ 14. The square of this number is twice the sum of 5 and 13.
- _____ 15. Twice the square of this number is $\frac{1}{2}$ the square of 10.
- _____ 16. The square of 6 plus this number is equal to the square of 7.
- _____ 17. The square of this number is 8 less than $\frac{1}{2}$ the square of 12.
- _____ 18. This number, when multiplied by 3, is equal to 4 times the square of 3.
- _____ 19. The product of this number and 10 is equal to the square of this number.
- _____ 20. The sum of the square of this number and this number is 20.



Scores:

- | | |
|--------------|---------------|
| 0-3, Hum Dum | 4-7, Oh Hum |
| 8-10, Hum | 11-15, Hum Ho |
| 16-18, Ho Ho | 19-20, Hi Ho |



Using Charts in Solving Problems (For use with Section 3-6)

By working through the steps in the problems below, you will gain skill in using charts to help solve problems.

Problem 1 Uncle Louis is three times as old as Michelle. In eight years their ages will total 100. How old are they now?

a. What does the problem ask you to find?

b. Suppose x represents Michelle's age now. Write an expression for Michelle's age 8 years from now.

c. Michelle is x years old now. Uncle Louis is three times as old as Michelle. Write an expression for Uncle Louis's age now.

d. Using the expression that you found in part c, write an expression for Uncle Louis's age 8 years from now.

e. Complete this chart using the expressions that you found in parts b, c, and d.

	Age now	Age in 8 years
Michelle	x	
Uncle Louis		

f. "In eight years their ages will total 100" means that

$$\begin{array}{l} \text{Michelle's age} \\ \text{in 8 years} \end{array} + \begin{array}{l} \text{Uncle Louis's} \\ \text{age in 8 years} \end{array} = 100$$

Write an equation that states this relationship.

g. Solve your equation for x . Then find $3x$.

h. Write your answer to the problem.

i. Check your results with the statement of the problem.

Is Uncle Louis three times as old as Michelle?

In eight years will their ages total 100?

(continued)

Using Charts in Solving Problems (continued)

Problem 2 Idaho became a state of the United States 40 years after California. In 1900, California had been a state five times as long as Idaho. How many years will California have been a state by 1990?

- a. Complete: The problem asks you to find the number of years that _____ will have been a state in the year _____.
- b. Let x = California statehood age in 1990. (Note that this is what you need to find.) Write an expression for California's age in 1900 (in terms of x). _____
- c. Remember that Idaho became a state 40 years *after* California did. Write an expression for Idaho's statehood age in 1990 (in terms of x). _____
- d. In terms of x , what was Idaho's age in 1900? _____
- e. Complete this chart using your answers in parts b, c, and d.

	Age in 1990	Age in 1900
California	x	
Idaho		

- f. The problem states that in 1900, California had been a state five times as long as Idaho. Use this statement and the chart to write an equation for the problem.
- _____
- g. Solve your equation. _____
- h. Write your answer to the problem. _____
- i. Check your results with the statement of the problem.

In 1900, California's age was _____ years and Idaho's age was _____ years. Do these values agree with the given facts of the problem? _____

How many years will California have been a state by 1990? _____

Is a Picture Worth a Thousand Words?

Match each graph with the correct equation.

_____ 1. $y = 4$

_____ 2. $y = \frac{1}{2}x + 1$

_____ 3. $y = x + 3$

_____ 4. $y = -2x + 4$

_____ 5. $y = 2x - 1$

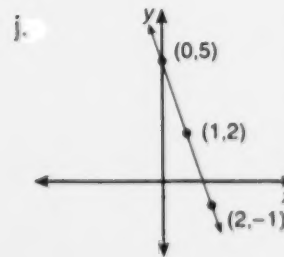
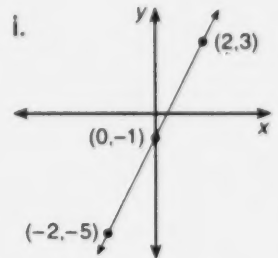
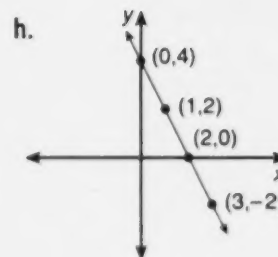
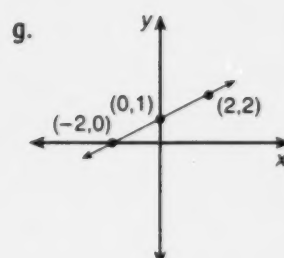
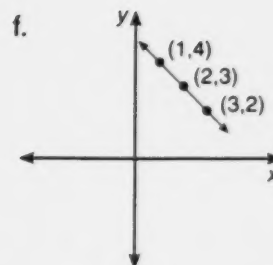
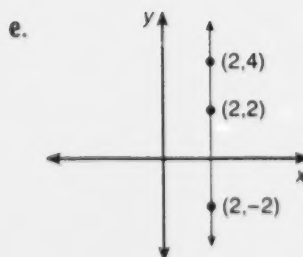
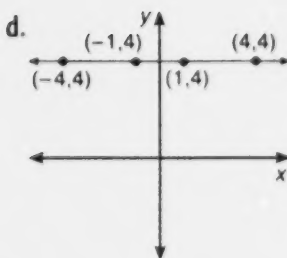
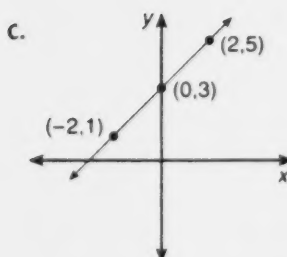
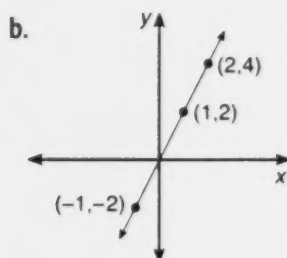
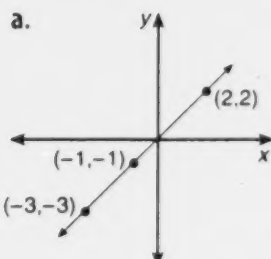
_____ 6. $y = x$

_____ 7. $x = 2$

_____ 8. $y = 5 - 3x$

_____ 9. $y = -x + 5$

_____ 10. $y = 2x$

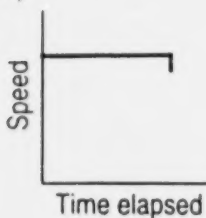


IDENTIFYING QUALITATIVE GRAPHS

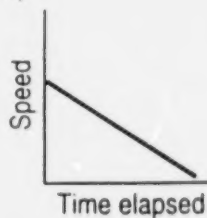
Indicate which graph matches the statement.

1. A train pulls into a station and lets off its passengers.

a)



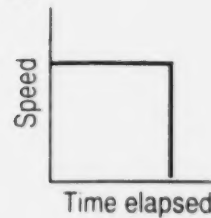
b)



c)

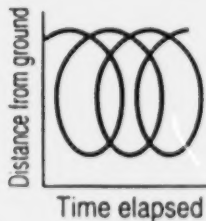


d)

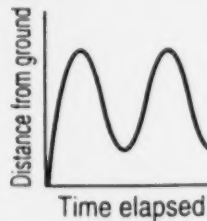


2. A man takes a ride on a ferris wheel.

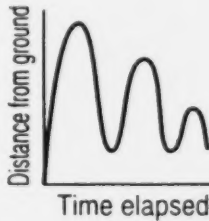
a)



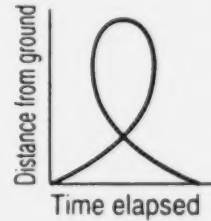
b)



c)

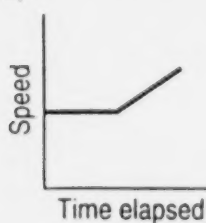


d)



3. A woman climbs a hill at a steady pace and then starts to run down one side.

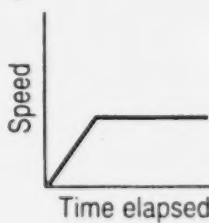
a)



b)



c)



d)



4. A child swings on a swing.

a)



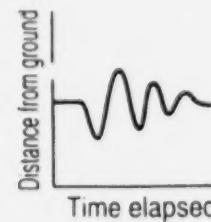
b)



c)



d)

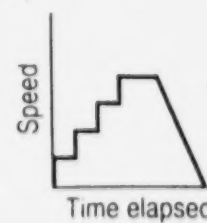


5. A child climbs up a slide and then slides down.

a)



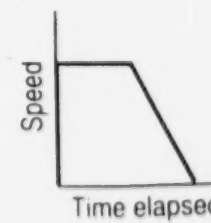
b)



c)



d)



Notes about graphs of equations in 2 unknowns

- 1)
 - We use x to represent the independent variable. The independent variable represents the information that you start with, the info that you choose or that is fixed. The independent variable might be the years, the months, the number of people, or anything you identify that you want to compare other information to.
 - We use y to represent the dependent variable. The value of the dependent variable depends on the value of the independent variable.
- 2) On a graph, x is represented on the horizontal axis and y is represented on the vertical axis.
- 3) The equation of the line can be stated in the form: $y = mx + b$, where m represents the slope of the line and b represents the y -intercept. (Y -intercept means the point at which the line crosses the y or vertical axis.)
- 4) To figure the slope of the line, choose 2 points on the line. Think of one of them as (x_1, y_1) and think of the other as (x_2, y_2) . Subtract $y_2 - y_1$ and $x_2 - x_1$. Then divide the first (or y) answer by the second (or x) answer. (I suggest using some of the space below to write an example of finding the slope.)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- 5) To find the equation of the line, including the y -intercept, choose one of your 2 points from figuring the slope and think of it as (x_1, y_1) . (It does not have to be the same point that you called (x_1, y_1) before. Put those values into the following equation for x_1 and y_1 ; then work through the equation to put it in the form $y = mx + b$.
(y = the slope times x + b)

$$m = \frac{y - y_1}{x - x_1}$$

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